Distributional robustness and inequity mitigation in disaster preparedness of humanitarian operations

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Abstract

We study a predisaster relief network design problem with uncertain demands. The aim is to determine the prepositioning and reallocation of relief supplies. Motivated by the call of the International Federation of Red Cross and Red Crescent Societies (IFRC) to leave no one behind, we consider three important practical aspects of humanitarian operations: shortages, equity, and uncertainty. We first employ a measure that we call the *Shortage Severity Measure* to evaluate the severity of the shortage caused by uncertain demand. Because shortages often raise concerns about equity, we then formulate a mixed-integer lexicographic optimization problem with non-convex objectives and propose a new branch-and-bound algorithm to identify the exact solution. We also propose two approaches for identifying optimal postdisaster adaptable resource reallocation: an exact approach and a conservative approximation that is more computationally efficient. Numerical studies on the 2010 Yushu earthquake positively demonstrate the value of our methodology in alleviating geographical inequities and reducing shortages. Our case study also provides several other interesting insights that may be useful for humanitarian organizations for disaster preparedness.

Key words: disaster preparedness, shortage, equity, distributionally robust optimization

1 Introduction

The number of disasters reported worldwide and their impact on the population has increased in recent decades. Extreme events such as tornadoes, earthquakes, or hurricanes can strike a community without warning and cause massive damage and many casualties. For example, the Emergency Events Database has recorded 7,348 natural disasters over the last twenty years (2000-2019), affecting over 4 billion people (many on more than one occasion), and causing economic losses of $2.97 trillion around the world (EM-DAT, 2020). The massive-scale social and economic damages caused by disasters have brought increasing attention to the need for effective disaster relief management.

Prepositioning of emergency supplies can be an effective mechanism for improving response to natural disasters (Salmerón and Apte, 2010). In the fall of 2019, hurricane Dorian was estimated to have caused up to $3 billion in losses in the Caribbean (CNBC, 2019), and highlighted the inadequacy of existing prepositioning strategies. This paper focuses on the prepositioning strategy in disaster relief systems and considers the Predisaster Relief Network Design Problem (PRNDP) to prepare for sudden disasters. This problem determines the locations and capacities of the response facilities and the inventory levels at each facility, as well as reallocations of relief supplies to distribution locations in order to improve the effectiveness of the postdisaster relief operations.

A good location and emergency inventory prepositioning strategy is critical for disaster relief operations, since lack of relief supplies may cause suffering and loss of life among victims. However, disaster preparedness is subject to considerable uncertainty because it is not known where events will occur (or if they will occur at all). As a result, disaster-stricken areas often face shortages of emergency supplies. For example, the Fritz Institute reported that there was a massive shortage of supplies and medical personnel during the 2004 tsunami in Southeast Asia (Fritz Institute, 2005). Food and water shortages also appeared in the Philippines after being hit by a super typhoon Haiyan in 2013 (Uichanco, 2021). Furthermore, in the winter of 2016/2017, an extreme dzud (a kind of dreaded severe weather) exposed more than 255,000 herders in Mongolia to water and food shortages and killed millions of animals (BBC, 2016). To alleviate the suffering caused by shortages, since 2011, appeals for funding by humanitarian organizations (HOs) have steadily increased, but more than 55% of the requirements have still not been met (Besiou and Van Wassenhove, 2020).

Shortages of relief supplies often raise concerns about the equity of disaster relief systems. Specifically, on 1 January 2016, the world officially began implementation of the 2030 Agenda for Sustainable Development and pledged to leave no one behind (UNSDG, 2019). In addition, many large international HOs, such as the IFRC, began to call on leaving no one behind in humanitarian response (IFRC, 2018). However, there is little research to-date on the equity regarding predisaster deployment decisions. We attempt to bridge this gap in our research.

1.1 Research Questions

This study mainly attempts to answer the following research questions:

- *How to measure the severity of possible shortages in the presence of demand uncertainty?* Predisaster deployment decisions are often constrained by demand uncertainty and limited budgets (Eftekhar et al., 2021), which may lead to shortages of supplies in certain affected areas after a disaster. The impact of the shortage varies from region to region. If a region has a strong industrial foundation, an adequate labor force, and advanced technology, it can quickly carry out postdisaster reconstruction to overcome a current shortage. However, some of the most vulnerable people in the world cannot bear the impact of temporary shortages in food and medicine. Therefore, humanitarian practitioners need to measure the severity of the shortage according to local conditions under demand uncertainty to make effective predisaster deployment decisions.

- *How to allocate limited resources equitably among beneficiaries to reduce the impact of shortages?* Equity is an essential requirement in humanitarian operations and has received widespread attention. Starr and Van Wassenhove, writing in the Special Issue for the Board of the POMS College on Humanitarian Operations and Crisis Management (Starr and Van Wassenhove, 2014), state "Humanitarians need to bring relief items to all beneficiaries in an equitable fashion, even if this is far from being efficient". They continue "There is an obvious need to consider equity in addition to classical efficiency or cost minimization objectives." Therefore, HOs need to give a more formal treatment of equity in predisaster deployment and humanitarian response (Besiou and Van Wassenhove, 2020).

- *How to address the inherent difficulty of quantifying postdisaster demand to improve on predisaster deployment decisions and postdisaster response decisions?* The initial prepositioning deployment decisions are difficult to make in the presence of demand uncertainty. For the HOs to deploy high inventory levels in each possible disaster area will be too costly for their limited budgets and donations, especially if postdisaster demands are relatively small (Stauffer and Kumar, 2021). If prepositioning of emergency supplies is not enough, then during a major disaster, many areas may suffer from shortages and even secondary casualties. Thus, HOs want to find a trade-off between small initial deployment levels that are within budget/donation constraints and large initial deployment levels that avoid serious shortages. Furthermore, a reasonable description of uncertain relief demands is a key issue for helping HOs find such a balance (Uichanco, 2021).

1.2 Our Contributions

1. *Model*: Our model considers three important practical aspects of humanitarian operations (i.e., shortages, equity, and uncertainty). First, to the best of our understanding, this is the first paper to use the *Shortage Severity Measure* (SSM) to control the uncertain relief shortages. Specifically, as an example of *satisficing measure* (Brown and Sim, 2009), the SSM is an axiomatically motivated way of measuring the severity of random shortages when compared to targeted maximum shortage thresholds. Second, we propose a model that ensures that the allocations of the limited resources are equitably distributed among disaster-prone regions. This is done by formulating a mixed-integer lexicographic optimization problem with non-convex objectives. Third, to account for the inherent difficulty of quantifying post-disaster demand distributions, we employ a two-stage robust stochastic optimization model that relies on scenario-wise moment information. Overall, we consider our model to take a important step toward bridging the gap between theory and practice in humanitarian operations.

2. *Solution approach*: First, we discuss two approaches for identifying optimal adaptable resource reallocation: an exact approach and a conservative approximation that is based on affine decision rules and allows us to solve instances of realistic size. Empirically, the latter also appears surprisingly accurate with a maximum measured optimality gap of 8%. Second, we handle the *lexicographic minimization* aspect of the model using a new branch-and-bound algorithm. This algorithm corrects for a deficiency found in the procedure proposed by Qi (2017). In fact, our algorithm appears to be the first iterative scheme with the guarantee of finding exact solutions to non-convex, mixed-integer, lexicographic optimization problems.

3. *Managerial insights*: Our case study involving a real earthquake case, provide three interesting insights. First, there is no conflict between ensuring that all beneficiaries have equitable access to relief resources and reducing the total shortage. Second, if equity is ignored, the shortages experienced by some beneficiaries may not be alleviated with an increase in donations. Third, disaster scenario-wise information, if properly segmented, can alleviate inequities caused by uncertain relief demands.

The remainder of the paper is organized as follows. In Section 2, we provide a brief literature review related to our research. In Section 3, we discuss three important practical aspects of humanitarian operations (i.e., shortages, equity, and uncertainty) and present the model. In Section 4, we describe the solution procedure for the proposed model. In Section 5, we perform several numerical studies. In Section 6, we present some valuable managerial insights and conclude the study. All proofs can be found in the appendix.

Notations We use boldfaced characters to represent vectors (e.g., $\boldsymbol{x} \in \mathbb{R}^n$). We use $|\mathcal{L}|$ to denote the cardinality of a set $\mathcal{L}$ and $(x)^+$ to denote $\max(x, 0)$. We use $\mathscr{P}_0(\mathbb{R}^n)$ to represent the set of all probability distributions on $\mathbb{R}^n$. A random variable, $\tilde{\mathbf{d}}$, is denoted with a tilde sign, and we use $\tilde{\mathbf{d}} \sim \mathbb{P}$, $\mathbb{P} \in \mathscr{P}_0(\mathbb{R}^n)$ to define $\tilde{\mathbf{d}}$ as a n-dimensional random variable with distribution $\mathbb{P}$. We assume that $\mathbb{P}$ lies in a distributional ambiguity set $\mathcal{F} \subset \mathscr{P}_0(\mathbb{R}^n)$ and denote $\mathbb{E}_{\mathbb{P}}[\cdot]$ as the expectation over the probability distribution $\mathbb{P}$.

2 Literature Review

In this section, we present a review of the related literature. Specifically, we focus on the following streams of literature that are related to our study: (i) PRNDP under uncertainty; (ii) supply shortages in humanitarian operations; and (iii) equity considerations. After that, we discuss the distinction between our paper and the existing literature.

2.1 Predisaster Relief Network Design

Predisaster relief network design involves decisions regarding facility locations, inventory prepositioning and resource reallocation under uncertainty. Every part of a decision can be viewed as an optimization problem that has gained considerable attention in the operations research and management science (OR/MS) communities. The PRNDP as a whole was first studied by Balcik and Beamon (2008), where the authors propose a scenario-based model to maximize the benefits provided to affected people. Rawls and Turnquist (2010) formally introduce the PRNDP that simultaneously determines the decisions of facility location, inventory prepositioning and resource distribution under demand uncertainty. They formulate a risk-neutral two-stage stochastic programming model and propose a Lagrangian L-shaped method. Following their path, stochastic programming is widely used to address uncertainty in PRNDP. In addition, some literature begins to focus on the measurement of risk by applying concepts such as the conditional value-at-risk (CVaR) (Noyan, 2012) and probabilistic constraints (Rawls and Turnquist, 2011; Hong et al., 2015). Elçi and Noyan (2018) further develop a chance-constrained two-stage stochastic programming model that combines quantitative risk (CVaR) and qualitative risk (probabilistic constraints).

All of the above literature is scenario-based stochastic programming, which has to deal with difficulties in selecting scenarios, especially in disaster management. This challenge motivates research on the robust optimization method as an alternative. Building on the pioneering work by Bertsimas and Sim (2004) on robust optimization, Ni et al. (2018) propose a min-max robust model that integrates the three-part decision making of predisaster relief network design. To address uncertainties in supply, demand and road link capacity, they construct budgeted uncertainty sets and develop computationally tractable reformulations based on the budgeted uncertainty sets. Similarly, Velasquez et al. (2019) apply the budgeted uncertainty set to model demand uncertainty and propose a robust model for prepositioning relief items. Paul and Wang (2019) further develop a two-stage robust optimization model and consider two types of robustness, one of which is the budgeted uncertainty set.

To hedge uncertainty in humanitarian operations, stochastic relief network design often assume uncertainty parameters following fully known distributions and use a finite number of scenarios to model uncertainties. This mainly faces two challenges in practical implementation: (i) computational difficulties for instances with abundant scenarios; (ii) sampling difficulties for uncertainty parameters of high dimensionality. On the other hand, although robust optimization with budgeted uncertainty sets does not require any knowledge of distributions except for their support, it tends to produce overly conservative solutions due to hedging against the rare worst case.

2.2 Shortage

To reduce shortages in disaster areas, the existing optimization models are mainly described from two perspectives: the objective function and the conditions on the constraints. As for the former, shortage is added to the objective function as a penalty cost. Most optimization models for predisaster relief network design use cost minimization as their objective to reduce shortages. Specifically, in the widely used two-stage stochastic and/or robust programming models, shortage cost is usually considered in the second stage, whereas facility cost and ordering cost are incurred in the first stage. From the perspective of constraint conditions, in order to control shortages, Rawls and Turnquist (2011) introduce *service quality* constraints that ensure that the probability of meeting all demand is not lower than a given target level. Salas et al. (2012) add a penalty cost when the shortage exceeds a threshold for two days in a row. Hong et al. (2015) apply probabilistic constraints to ensure that the demand in each region is satisfied.

Because the penalty cost of shortage is difficult to determine in practice, some HOs will not use cost to guide their decision. Gralla et al. (2014) study five attributes (e.g., quantity, type, location, speed, and cost) based on the preferences of experts toward humanitarian logistics, and the results show that quantity delivered is the most valued objective while cost is the least important. In addition, although we can ensure that the satisfaction of demands is maintained at a certain level by probabilistic constraints, we may ignore the unmet demands and the seriousness of their impact.

2.3 Equity

To address equity considerations in humanitarian operations, it is necessary to clarify the exact meaning of equity. A number of researchers have analyzed the concept of equity in detail, see Savas (1978) and Fishburn and Straffin (1989). Although equity is a very important consideration in the literature, there is no definite consensus on its definition. In the fields of OR/MS, there are many different ways of incorporating equitability in the decision making process. For a comprehensive overview of equity in OR/MS problems, we refer readers to the recent reviews provided by Karsu and Morton (2015). One of the most common ways of modeling equity is the lexicographic optimization approach, which is applied in many fields, such as resource allocation (Luss, 1999), network flows (Nace and Orlin, 2007) and appointment scheduling (Qi, 2017). According to Karsu and Morton (2015), the solution derived by the lexicographic approach is sometimes considered to be the *most equitable* solution.

In humanitarian operations, equity concerns play a role in the disaster response phase in the reallocation of relief items to beneficiaries. To provide relief resources to demand locations in a fair manner, the existing literature mainly studies equity from the perspectives of response time and supply quantity. Holguín-Veras et al. (2013) introduce a new concept called *deprivation cost*, which depends on the deprivation time. They quantify equity concerns for postdisaster humanitarian logistics by using social costs, which is the sum of logistic and deprivation costs. Following their direction, Ni et al. (2018) measure the shortage cost associated with unsatisfied demand through the deprivation cost to ensure the equitable relief delivery operations in some sense. Gutjahr and Fischer (2018) show that minimizing the total deprivation cost given a budget may yield inequitable solutions and they propose to extend the deprivation cost objective with the Gini inequity index. In terms of equitable allocation quantities, Noyan et al. (2016) compute the *maximum proportion of unsatisfied demand* among demand locations and apply a proportional allocation policy to ensure equitable allocation of resources in the last mile. Velasquez et al. (2019) introduce equity constraints to ensure that relief items are distributed proportionately to the demand. Arnette and Zobel (2019) apply a measure of relative risk and develop a risk-based objective function to ensure equitable allocations of assets in advance of a natural disaster. Recently, Uichanco (2021) propose a stochastic programming model for typhoon preparedness with two objectives, one of which is a fair strategy by minimizing the expected largest proportion of unmet demand.

2.4 Distinction of Our Work from Past Literature

Motivated by special features that HOs face in practice, our paper considers three important aspects of humanitarian operations: shortages, equity, and uncertainty. Previous studies on predisaster relief network design mainly use a cost criteria (e.g., setup cost, purchase cost, transportation cost, shortage cost, etc.), while some HOs will not use cost to guide their decision (Uichanco, 2021). Besiou and Van Wassenhove (2020) also argue that it is not easy to evaluate the performance of humanitarian operations through cost. Furthermore, as we mentioned before, HOs usually put the cost in the least important position and the amount of supply delivered in the first place (Gralla et al., 2014). Therefore, we use the amount of supply shortage to measure the severity of the shortage and formulate a mixed-integer lexicographic optimization problem with non-convex objectives.

Although there has been significant progress in addressing lexicographic optimization models (see, for instance, Marchi and Oviedo, 1992; Nace and Orlin, 2007), these approaches cannot be applied to non-convex models with discrete and continuous decisions, which have received less attention in the literature. Nace and Orlin (2007) provide a polynomial approach for linear lexicographic optimization problems and prove its optimality. Ogryczak et al. (2005) propose a reformulation based on conditional means for lexicographic optimization problems with non-convex feasible sets. Since our model has a non-convex objective function, this method will lead to a mixed-integer non-convex programming reformulation, which we expect to quickly become intractable for large-scale problems. Recently, Letsios et al. (2021) propose a branch-and-bound algorithm for a scheduling problem to obtain exact lexicographic scheduling. Because the scheduling problem belongs to a typical combinatorial optimization problem, they enumerate all possible job-to-machine assignment, which cannot be applied to problems that include both discrete and continuous decisions. In this stream of literature, the most relevant work to ours is Qi (2017), who propose a lexicographic minimization procedure for mixed-integer lexicographic optimization model with non-convex objectives. However, we find that such a procedure cannot guarantee optimality for our problems. Therefore, our paper corrects for a deficiency found in Qi (2017). Specifically, we propose a new branch-and-bound algorithm for solving non-convex, mixed-integer, lexicographic optimization problems and prove its optimality.

3 Problem Formulation

In this section, we first present a multi-objective two-stage stochastic optimization model for the PRNDP. We then introduce a measure to evaluate shortages in the presence of demand uncertainty. Finally, we formulate a lexicographic minimization problem to address equity concerns.

3.1 A Two-stage Model

Given a set of potential demand locations $\mathcal{L}$, we assume that a facility can be opened at any such demand locations. Each location $i \in \mathcal{L}$ represents a geographical area (e.g., state, county, district, etc.) with a random relief demand $\tilde{d}_i : \Omega \to \mathbb{R}_+$ on a probability space $(\Omega, \Sigma, \bar{\mathbb{P}})$. Let $\mathcal{K}$ denote the set of possible facility sizes (i.e., capacity limits), indexed by κ, and let $M_\kappa > 0$ denote the maximum capacity of facilty of category κ. Associated with each candidate location i and each size category κ is a fixed location cost $c_{i\kappa} > 0$. We consider a single type of inventory unit that consists of a bundle of critical relief supplies, including prepackaged food, medical kits, blankets, and water. The total amount of these emergency supplies, denoted by $R > 0$, is determined by the donations received. Let $B > 0$ denote the total budget for the predisaster deployment. In the predisaster operations, HOs need to decide where to set up the facilities for prepositioning emergency supplies and how much inventory to preposition in each facility that has been opened. After a disaster, the reallocation operation of emergency supplies should be able to adjust adaptively. To formulate the model, let $x_{i\kappa} \in \{0, 1\}$ denote whether or not a facility of size category $\kappa \in \mathcal{K}$ is opened at location $i \in \mathcal{L}$ and let $r_i \geq 0$ denote the number of supplies prepositioned at location $i \in \mathcal{L}$. In addition, let $\tilde{y}_{ij} : \Omega \to \mathbb{R}_+$ be an adaptive strategy indicating the amount of supplies reallocated to location $j \in \mathcal{L}$ from location $i \in \mathcal{L}$ under each possible outcome $\omega \in \Omega$ and similarly $\tilde{u}_i : \Omega \to \mathbb{R}_+$ be a random variable denoting the planned amount of unsatisfied demand at location $i \in \mathcal{L}$. The parameters and decision variables for the model are summarized in Table 1.

Table 1: Model Parameters and Decision Variables

Sets	
$\mathcal{L}$	Set of demand locations
$\mathcal{K}$	Set of facility size categories
Parameters	
$c_{i\kappa}$	Fixed cost of opening a facility of size category κ at location i
M_κ	Capacity limit of a facility of size category κ
R	A total amount of emergency supplies
B	Budget limit for prepositioning supplies
τ_i	The tolerance threshold of supply shortage for demand location i
$\tilde{d}_i$	Random relief demand at location i
Decision variables	
$x_{i\kappa}$	1, if a facility of size category κ is opened at location i; 0 otherwise
r_i	The amount of supplies prepositioned at facility location i
$\tilde{y}_{ij}$	The amount of supplies allocated to location j from location i
$\tilde{u}_i$	The amount of unmet demand (supply shortage) at location i

To determine predisaster deployment decisions, we formulate the following constraints:

$$\sum_{\kappa\in\mathcal{K}} x_{i\kappa} \leq 1, \qquad i \in \mathcal{L}, \tag{1a}$$

$$r_i \leq \sum_{\kappa\in\mathcal{K}} M_\kappa x_{i\kappa}, \qquad i \in \mathcal{L}, \tag{1b}$$

$$\sum_{i\in\mathcal{L}} r_i \leq R, \tag{1c}$$

$$\sum_{i\in\mathcal{L}} \sum_{\kappa\in\mathcal{K}} c_{i\kappa} x_{i\kappa} \leq B. \tag{1d}$$

According to constraint (1a), not more than one facility can be opened at any demand location. Constraint (1b) states that the quantity of prepositioned relief items cannot exceed facility capacity. Constraint (1c) specifies that the total amount of the emergency supplies is R. Constraint (1d) ensures that the construction of response facilities are within the given budget. Note that one also can decide whether to consider other costs in the budget constraint according for different settings. Furthermore, the shortage can be represented by a convex piecewise linear function as defined next.

Definition 1. ***(Supply Shortage)*** *For any fixed decision $(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y})$ and realization $\mathbf{d}$, the relief supply shortage for location $i \in \mathcal{L}$ is defined by the function*

$$f(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y}, \mathbf{d}) := \left(d_i + \sum_{j\in\mathcal{L}\setminus i} y_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i} y_{ji}) \right)^+ .$$

This gives rise to the following multiobjective optimization problem under uncertainty:

$$(\mathrm{P_{MOU}}) \qquad \underset{\boldsymbol{x},\boldsymbol{r},\tilde{\boldsymbol{y}},\tilde{\boldsymbol{u}}}{\text{minimize}} \quad \{\tilde{u}_i\}_{i\in\mathcal{L}} \tag{2a}$$

$$\text{s.t.} \quad \tilde{u}_i = \left(\tilde{d}_i + \sum_{j\in\mathcal{L}\setminus i} \tilde{y}_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i} \tilde{y}_{ji}) \right)^+, \qquad \text{a.s., } i \in \mathcal{L}, \tag{2b}$$

$$\sum_{j\in\mathcal{L}\setminus i} \tilde{y}_{ij} \leq r_i, \qquad \text{a.s., } i \in \mathcal{L}, \tag{2c}$$

$$\tilde{y}_{ij} \geq 0, \qquad \text{a.s., } i, j \in \mathcal{L}, \tag{2d}$$

$$(1a) - (1d),$$

where “a.s.” stands for almost surely. Constraint (2b) ensures the exact calculation of the relief supply shortages. Constraint (2c) restricts that the amount of supply delivered from each facility does not exceed the level of prepositioned supplies. Note that the objective of problem $\mathrm{P_{MOU}}$ is both multiple, as it attempts to minimize the shortage at each location $i \in \mathcal{L}$, and uncertain, as the shortages depend on random demand. Hence, in the next two sections, we will discuss how we propose to control the risk of excessive shortage and trading off between the different locations in an equitable way. In doing so, it will be useful to summarize problem $\mathrm{P_{MOU}}$ using:

$$\underset{\tilde{\boldsymbol{u}} \in \mathscr{U}}{\text{minimize}} \quad \tilde{\boldsymbol{u}} \,,$$

where $\mathscr{U}$ represents the set of random shortage vectors that can be produced in problem $\mathrm{P_{MOU}}$, i.e.

$$\mathscr{U} := \{\tilde{\boldsymbol{u}} \mid \exists \boldsymbol{x}, \boldsymbol{r}, \tilde{\boldsymbol{y}}, (1\mathrm{a}) - (1\mathrm{d}), (2\mathrm{b}) - (2\mathrm{d})\}.$$

3.2 Shortage Severity Measure

For any feasible decisions $(\boldsymbol{x}, \boldsymbol{r})$, we can first consider in isolation how to treat the uncertainty about supply shortages $\tilde{u}_i$ at each location i. This will be done by assuming throughout this section that $\mathcal{L} = 1$, so that $\boldsymbol{u}$ will be referred as the random shortage $\tilde{u}$. The classical stochastic relief network design approaches assume a known probability setting and employ a risk measure such as the expected supply shortage (see, for instance, Rawls and Turnquist, 2010). One the other hand, classical robust optimization tends to produce overly conservative solutions because of ignoring any knowledge regarding the distribution except for its support. Therefore, we apply an alternative modeling paradigm known as distributionally robust optimization (DRO) and assume that $\mathbb{P}$ is only known to belong to a convex ambiguity set $\mathcal{F}$ that is characterized by partial distribution information estimated from historical data. As a result, we can seek the worst-case distribution to protect the risk measure by hedging against all probability distributions in $\mathcal{F}$. In our model, we will consider a generic risk constraint in the form

$$\sup_{\mathbb{P} \in \mathcal{F}} \rho\left(\tilde{u}\right) \leq \tau,$$

where $\rho(\tilde{u})$ denotes a risk measure of supply shortages, τ represents a bounding threshold for the risk of supply shortages.

Next, we discuss the specific form of risk measure. When the coherent risk measure CVaR is specified as the risk measure, we can set $\rho\left(\tilde{u}\right) := \mathrm{CVaR}_{1-\alpha}\left(\tilde{u}\right)$. We refer interested readers to Rockafellar et al. (2000a) and references therein for more details and examples of modeling and optimization problems using CVaR. In this setting, the CVaR is the expected shortage given that it falls beyond its $1 - \alpha$ quantile. Hence, intuitively, this setting ensures that the CVaR of the supply shortage with $1 - \alpha$ confidence remains under τ for all distributions in the set $\mathcal{F}$. In addition, CVaR has been adopted as a preferred measure in disaster management. For example, Elçi and Noyan (2018) suggest that CVaR is a reliable measure for the supply shortages. Alem et al. (2016) compare different risk measures, showing that the CVaR concept leads to higher demand satisfaction.

With the worst-case CVaR as a risk measure, one can impose

$$\sup_{\mathbb{P} \in \mathcal{F}} \mathrm{CVaR}_{1-\alpha}\left(\tilde{u}\right) \leq \tau \tag{3}$$

to keep the worst-case CVaR of the supply shortages below a threshold τ. Note that the worst-case CVaR is still a coherent risk measure and exhibits some valuable properties (Zhu and Fukushima, 2009). We propose an equivalent representation of the worst-case CVaR of shortages in Lemma 1.

Lemma 1. *The bounded worst-case CVaR constraint* (3) *is equivalent to:*

$$\inf_{\eta \geq 0} \left(\eta + \frac{1}{\alpha} \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right) \leq \tau. \tag{4}$$

Proof. Please see Appendix B.1.

With the worst-case CVaR of shortages in hand, we next introduce a measure to evaluate the uncertain relief supply shortage in Definition 2.

Definition 2. ***(Shortage Severity Measure)*** *Assume an uncertain supply shortage denoted by the random variable* $\tilde{u} \in \mathscr{U}$ *and a tolerance threshold* $\tau \geq 0$*. If we only know that the true distribution* $\mathbb{P}$ *lies in a distributional ambiguity set* $\mathcal{F}$*, we define the SSM* $\rho_\tau : \mathscr{U} \to [0, 1]$ *as follows:*

$$\rho_\tau(\tilde{u}) := \begin{cases} \inf\limits_{\alpha \in (0,1] : \mathcal{V}_{1-\alpha}(\tilde{u}) \leq \tau} \alpha & \textit{if } \mathcal{V}_0(\tilde{u}) \leq \tau, \\ 1 & \textit{if } \mathcal{V}_0(\tilde{u}) > \tau, \end{cases}$$

where $\mathcal{V}_{1-\alpha}(\tilde{u})$ *is the worst-case CVaR*$_{1-\alpha}$ *of the* $\tilde{u}$ *defined as*

$$\mathcal{V}_{1-\alpha}(\tilde{u}) := \inf_{\eta \geq 0} \left\{ \eta + \frac{1}{\alpha} \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right\}, \ \alpha \in (0, 1],$$

so that $\mathcal{V}_0(\tilde{u}) := \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}}[\tilde{u}]$.

Intuitively, for any given random shortage, SSM quantifies the risk of excessive shortage using a number between 0 and 1. If there is no possibility of shortage beyond τ, then the value of the SSM is 0. When the supply shortage is on average close or beyond τ, the SSM value will be close to 1. Moreover, the SSM can be regarded as the smallest upper 100α percentile, such that the worst-case CVaR$_{1-\alpha}$ of the supply shortage will not exceed the tolerable threshold τ. It is desirable to have an uncertain supply shortage with the smallest SSM value, because it implies that even for those unlikely massive demands, their worst-case CVaR can still be no more than the tolerance threshold. To the best of our knowledge, this is the first time that a measure that can account for the uncertain relief shortages in a mathematically precise way has been used in disaster management.

We note that the SSM falls within the framework of *satisficing measures* proposed by Brown and Sim (2009) and is analogous to the *Delay Unpleasantness Measure* in Qi (2017) and the *buffered probability of exceedance* in Mafusalov and Uryasev (2018). Specifically, SSM can be regarded as $\rho_\tau(\tilde{u}) := 1 - S(\tau - \tilde{u})$, where S is a satisficing measure. We further present several important properties in Proposition 1.

Proposition 1. *Given* $\tilde{u}, \tilde{u}_1, \tilde{u}_2 \in \mathscr{U}$*, the SSM satisfies the following properties:*

i) Monotonicity: if $\mathbb{P}(\tilde{u}_1 \leq \tilde{u}_2) = 1$ *for all* $\mathbb{P} \in \mathcal{F}$*, then* $\rho_\tau(\tilde{u}_1) \leq \rho_\tau(\tilde{u}_2)$.

ii) Satisfaction: $\rho_\tau(\tilde{u}) = 0$ *if and only if* $\mathbb{P}(\tilde{u} \leq \tau) = 1$ *for all* $\mathbb{P} \in \mathcal{F}$.

iii) Dissatisfaction: if $\sup\limits_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}}[\tilde{u}] > \tau$*, then* $\rho_\tau(\tilde{u}) = 1$.

iv) Quasi-convexity: for any $\hat{\alpha} \in (0, 1]$*, the set* $\mathscr{U}(\hat{\alpha}) := \{\tilde{u} | \rho_\tau(\tilde{u}) \leq \hat{\alpha}\}$ *is closed and convex.*

Proof. Please see Appendix B.2.

The *monotonicity* property implies that the smaller the shortage, the lower the risk. This is consistent with disaster management practices because the decision maker prefers smaller shortages. The *satisfaction* property ensures that, if an uncertain shortage is fully tolerable in the affected area, then the value of the measure is zero. This property also indicates that shortages below the tolerable threshold are the most preferred. The *dissatisfaction* property shows that, if an uncertain shortage exceeds the tolerable threshold of the affected area in worst-case expectation, then the severity of the shortage reaches the limit, which should be avoided in disaster management. Intuitively, when the uncertain shortage $\tilde{u}$ is closer to the threshold τ, the SSM value is closer to one. The *quasi-convexity* is an attractive property for optimization, which under the right conditions, which can include that $\mathcal{U}$ be convex, enables us to efficiently obtain a global SSM minimizer.

3.3 Equity Modeling

Equity is an essential issue in humanitarian operations and providing relief supplies to every affected individual in an equitable manner has become a social consensus. While there are many concepts with regard to equity, typically they can be divided into *horizontal* equity and *vertical* equity (see, for instance, Karsu and Morton, 2015). In the context of disaster management, *horizontal* equity refers to every individual or group being given the exact same resources to meet their needs, while *vertical* equity allocates the resources based on the different needs of the recipients. In Figure 1(a), for example, two identical medical kits are given to three people of different heights—it's an equal allocation of resources (*horizontal* equity), but it fails to consider that the tallest person does not need a kit to overcome the deprivations caused by a disaster, while the shortest person could clearly use an extra one. When the kits are redistributed equitably, according to *vertical* equity, as seen in Figure 1(b), all three individuals can survive the disaster.

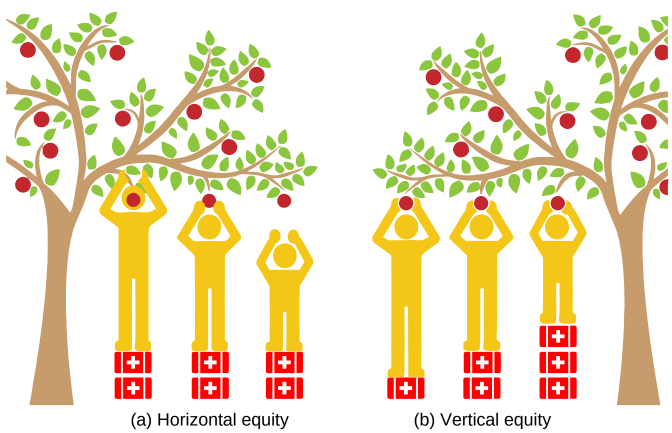


Figure 1: Horizontal Equity and Vertical Equity. (Source: this figure was adapted from a figure © 2014, Saskatoon Health Region.)

In humanitarian operations, since HOs need to consider relief demands of all victims from a global perspective and provide treatment accordingly, we mainly focus on *vertical* equity. Moreover, supply shortages often raise concerns about equity, so we characterize the concepts of an equitable solution (see, for instance, Luss, 1999) based on *vertical* equity from the perspective of shortages.

Definition 3. ***(Equitable Solution)*** *A solution is called equitable if no affected area can reduce its SSM value without raising an already equal or higher SSM measure of another affected area.*

To obtain such an equitable solution, we can formulate a lexicographic minimization problem according to the Rawlsian principle of justice (Rawls, 1971), as follows:

$$(\mathrm{P_{LM}}) \quad \tilde{\boldsymbol{u}}^* \in \arg \operatorname*{leximin}_{\tilde{\boldsymbol{u}} \in \mathscr{U}} \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}), \tag{5}$$

where $\boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}) := \left[\rho_{\tau_1}(\tilde{u}_1), \rho_{\tau_2}(\tilde{u}_2), ..., \rho_{\tau_{|\mathcal{L}|}}(\tilde{u}_{|\mathcal{L}|})\right]^\top$. Let $\tilde{\boldsymbol{u}}^*$ denote the optimal lexicographic solution that provides the lexicographically minimal vector $\boldsymbol{\alpha}^* := \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}^*)$. To keep this paper self-contained, we briefly provide the definition of lexicographic order in Definition 4.

Definition 4. ***(Lexicographic Order)*** *Given $\boldsymbol{\delta} \in \mathbb{R}^{|\mathcal{L}|}$, let $\vec{\boldsymbol{\delta}} \in \mathbb{R}^{|\mathcal{L}|}$ denote the vector $\boldsymbol{\delta}$ with its indices reordered so that the components are in nonincreasing order. The vector $\boldsymbol{\delta} \in \Delta$ is lexicographically less than $\boldsymbol{\delta}' \in \Delta$, denoted by $\boldsymbol{\delta} \preceq \boldsymbol{\delta}'$ if either $\vec{\boldsymbol{\delta}} = \vec{\boldsymbol{\delta}}'$ or there exists a $k \in \{1, \ldots, |\mathcal{L}|\}$, such that $\vec{\boldsymbol{\delta}}_i = \vec{\boldsymbol{\delta}}'_i$ for all $i < k$ and $\vec{\boldsymbol{\delta}}_k < \vec{\boldsymbol{\delta}}'_k$. Furthermore, a vector $\boldsymbol{\delta} \in \Delta$ is said to be lexicographically minimal in some $\Delta \subseteq \mathbb{R}^{|\mathcal{L}|}$ if for every vector $\boldsymbol{\delta}' \in \Delta$, $\boldsymbol{\delta} \preceq \boldsymbol{\delta}'$.*

Remark 1. *In problem P_{LM}, we note that our definition of lexicographic ordering does not impose an a-priori ordering on which elements of $\boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}})$ should be minimized first. It rather should be understood as minimizing in order from 1 to $|\mathcal{L}|$ the terms of the sorted (in decreasing order) $\vec{\boldsymbol{\rho}}_{\boldsymbol{\tau}}(\tilde{\boldsymbol{u}})$.*

With this objective in hand, we can present our proposed predisaster relief network design problem with equity (PRNDP-E):

$$\text{(PRNDP-E)} \qquad \operatorname*{leximinimize}_{\boldsymbol{x}, \boldsymbol{r}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{u}}} \quad \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}) \tag{6a}$$

$$\text{s.t.} \quad (1a) - (1d), (2b) - (2d). \tag{6b}$$

where "leximinimize" refers to our search for the minimal feasible solution in terms of lexicographic ordering. Mathematically, $\boldsymbol{x}$ is discrete while $\boldsymbol{r}$ is continuous, both $\tilde{\boldsymbol{y}}$ and $\tilde{\boldsymbol{u}}$ are adaptable, and each $\rho_\tau(\cdot)$ is quasi-convex, we say that our problem belongs to the class of distributionally robust mixed-integer non-convex two-stage lexicographic optimization problems. Furthermore, as we show in Section 2.4 and Appendix A, we find that the *lexicographic minimization procedure* proposed by Qi (2017) for such problems cannot guarantee optimality. This motivates us to develop an efficient computational method with the guarantee of finding exact solutions to non-convex, mixed-integer, lexicographic optimization problems.

4 Solution Approach

In this section, we first propose a new branch-and-bound algorithm to address the *lexicographic minimization* aspect of the model PRNDP-E. We then handle demand–distribution ambiguity by the robust stochastic optimization approach. Finally, we discuss two approaches for identifying optimal adaptable resource reallocation.

4.1 The Branch-and-Bound Algorithm

We first focus on proposing a branch-and-bound algorithm (see Algorithm 1) for solving non-convex mixed-integer lexicographic optimization models of the form presented in problem $\mathrm{P_{LM}}$. For convenience, we refer to the lexicographic minimal vector as $\boldsymbol{\alpha}^* := \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}}^*) \in \mathbb{R}^{|\mathcal{L}|}$, to the lexicographic order as $\preceq$, and to $\bar{\boldsymbol{\alpha}}$ and $\underline{\boldsymbol{\alpha}}$ as respective upper and lower bounds for $\boldsymbol{\alpha}^*$ if $\underline{\boldsymbol{\alpha}} \preceq \boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}$. The algorithm starts by minimizing a worst-case SSM over all locations $i \in \mathcal{L}$. Then the procedure branches according to locations imposing that the SSM for this location does not exceed the minimax value that was identified at its parent node. Hence, each node n in the enumeration tree $\mathscr{N}$ corresponds to a minimax problem (7) in which the SSM for some locations in $\bar{\mathcal{L}}^n$

cannot exceed $\underline{\alpha}_i^n$, while the worst SSM is minimized for the other locations (i.e., $i \in \mathcal{L}/\bar{\mathcal{L}}^n$). A lower bound $\underline{\boldsymbol{\alpha}}^n$ for the children of each node is also maintained and compared to the best solution found $\bar{\boldsymbol{\alpha}}$ so far in order to trim the node if no improvement can be achieved.

Algorithm 1 Branch and Bound Algorithm for Lexicographic Optimization Problem $\mathrm{P_{LM}}$

1: Input: A set $\mathcal{U}$ and vectored risk measure $\boldsymbol{\rho} : \mathcal{U} \to \mathbb{R}^{|\mathcal{L}|}$

2: Output: The lexicographic minimal vector $\boldsymbol{\alpha}^*$ and a lexicographic minimal solution $\boldsymbol{u}^*$.

3: Set $\bar{\alpha}_i := \infty$ for all i, and some arbitrary $\bar{\boldsymbol{u}} \in \mathcal{U}$.

4: Initialize enumeration tree $\mathscr{N} := \{n_0\}$ with $\bar{\mathcal{L}}^{n_0} := \mathcal{L}$, $\underline{\boldsymbol{\alpha}}_i^{n_0} := -\infty$ for all i.

5: **while** $\mathscr{N} \neq \emptyset$ **do**

6: Select a node n in the enumeration tree $\mathscr{N}$ and remove n from $\mathscr{N}$ (*Node selection*)

7: **if** $\underline{\boldsymbol{\alpha}}^n \prec \bar{\boldsymbol{\alpha}}$ **then** (*Node expansion*)

8: Solve the following minimax problem associated with node n,

$$\min_{\tilde{\boldsymbol{u}} \in \mathscr{U}} \max_{i \in \mathcal{L}/\bar{\mathcal{L}}^n} \rho_{\tau_i}(\tilde{u}_i) \tag{7a}$$

$$\text{s.t. } \rho_{\tau_i}(\tilde{u}_i) \leq \underline{\alpha}_i^n, \ i \in \bar{\mathcal{L}}^n, \tag{7b}$$

9: Let v_n^* and $\tilde{\boldsymbol{u}}_n^*$ be optimal value and minimizer of problem (7)

10: Construct $\bar{\boldsymbol{\alpha}}^n$ as follows:

$$\bar{\boldsymbol{\alpha}}_i^n := \begin{cases} \underline{\alpha}_i^n & \text{if } i \in \bar{\mathcal{L}}^n \\ v_n^* & \text{otherwise} \end{cases}$$

11: **if** $\bar{\boldsymbol{\alpha}}^n \preceq \bar{\boldsymbol{\alpha}}$ **then** (*Update best solution*)

12: Let $\bar{\boldsymbol{\alpha}} := \bar{\boldsymbol{\alpha}}^n$ and $\bar{\boldsymbol{u}} := \tilde{\boldsymbol{u}}_n^*$.

13: **end if**

14: **if** $|\mathcal{L}/\bar{\mathcal{L}}^n| > 1$ **then** (*Branching*)

15: **for** $j \in \mathcal{L}/\bar{\mathcal{L}}^n$ **do**

16: Create new node n' with $\bar{\mathcal{L}}^{n'} := \bar{\mathcal{L}}^n \cup j$ and

$$\underline{\alpha}_i^{n'} := \begin{cases} \underline{\alpha}_i^n & \text{if } i \in \bar{\mathcal{L}}^n \\ v_n^* & \text{if } i = j \\ -\infty & \text{otherwise} \end{cases}$$

17: Append the new node n' to $\mathscr{N}$.

18: **end for**

19: **end if**

20: **end if**

21: **end while**

22: **return** $\boldsymbol{\alpha}^* := \bar{\boldsymbol{\alpha}}$ and $\boldsymbol{u}^* := \bar{\boldsymbol{u}}$.

Remark 2. *In the step of branching (i.e., Step* (16)*), one can alternatively obtain a lower bound using for all $i \in \mathcal{L}/\bar{\mathcal{L}}^{n'}$ by*

$$\begin{aligned} \underline{\alpha}_i^{n'} := \min_{\tilde{\boldsymbol{u}} \in \mathscr{U}} \quad & \rho_{\tau_i}(\tilde{u}_i) \\ s.t. \quad & \rho_{\tau_j}(\tilde{u}_j) \leq \underline{\alpha}_j^n, \ j \in \bar{\mathcal{L}}^n, \\ & \rho_{\tau_j}(\tilde{u}_j) \leq v_n^*, \ j \in \mathcal{L}/\bar{\mathcal{L}}^n. \end{aligned}$$

Remark 3. *In practice, different strategies can be used to select the node n in Step* (6)*. In our implementation, we select the node with lowest lower bound first.*

We also prove the optimality of the output vector $\boldsymbol{\alpha}^*$ in Theorem 1.

Theorem 1. *The vector $\boldsymbol{\alpha}^*$ returned by Algorithm 1 is lexicographically minimal, i.e. $\boldsymbol{\alpha}^* \preceq \boldsymbol{\rho_\tau}(\tilde{\boldsymbol{u}})$ for all $\tilde{\boldsymbol{u}} \in \mathscr{U}$.*

Proof. Please see Appendix B.3.

Since we need to solve a series of similar minimax problems (7) repeatedly, we then mainly focus on solving these problems. By Definitions 1 and 2, we can reorganize problem (7) as the following distributionally robust optimization (DRO) problem:

$$
\begin{aligned}
(\mathrm{P_{DRO}})\ v_n^* := \inf_{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\tilde{\boldsymbol{y}},\tilde{\boldsymbol{u}}} \ & \alpha && && \text{(8a)}\\
\text{s.t.}\ \ & \eta_i + \frac{1}{\alpha}\sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[(\tilde{u}_i - \eta_i)^+\right] \le \tau_i, && i \in \mathcal{L}\setminus\bar{\mathcal{L}}^n, && \text{(8b)}\\
& \eta_i + \frac{1}{\underline{\alpha}_i^n}\sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}}\left[(\tilde{u}_i - \eta_i)^+\right] \le \tau_i, && i \in \bar{\mathcal{L}}^n, && \text{(8c)}\\
& \tilde{u}_i = \left(d_i + \sum_{j\in\mathcal{L}\setminus i}\tilde{y}_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i}\tilde{y}_{ji})\right)^+, && \text{a.s. under all } \mathbb{P}\in\mathcal{F},\ i\in\mathcal{L}, && \text{(8d)}\\
& \sum_{j\in\mathcal{L}\setminus i}\tilde{y}_{ij} \le r_i, && \text{a.s. under all } \mathbb{P}\in\mathcal{F},\ i\in\mathcal{L}, && \text{(8e)}\\
& \tilde{y}_{ij} \ge 0, && \text{a.s. under all } \mathbb{P}\in\mathcal{F},\ i,j\in\mathcal{L}, && \text{(8f)}\\
& \text{(1a)} - \text{(1d)}, \alpha \in (0,1], \boldsymbol{\eta} \ge \mathbf{0}. && && \text{(8g)}
\end{aligned}
$$

We note that $\eta_i + \dfrac{1}{\alpha}\sup_{\mathbb{P}\in\mathcal{F}}\mathbb{E}_{\mathbb{P}}[\tilde{u}_i - \eta_i]^+$ is non-increasing in α which implies that the set of feasible α's has the form $[\alpha^*, 1]$. Hence, one can solve problem $\mathrm{P_{DRO}}$ by performing a bisection search for α^*, which at each step tests for the feasibility $\mathrm{P_{DRO}}$ when α is fixed to some value. The latter reduces to verifying the feasibility of a convex optimization problem. However, we still face two challenges: (i) how to construct an ambiguity set $\mathcal{F}$; and (ii) how to develop a tractable formulation so that a feasible solution can be quickly found. Hence, in the subsections below, we describe the solution procedure for the problem $\mathrm{P_{DRO}}$.

4.2 The Robust Stochastic Optimization Approach

To handle problem $\mathrm{P_{DRO}}$, we deploy the framework of robust stochastic optimization (RSO) (Chen et al., 2020), where the uncertainty associated with problems is described by a scenario-wise ambiguity set.

4.2.1 Scenario-wise Ambiguity Set.

The uncertain demands are strongly correlated with covariates (e.g., the Seismic magnitude scales, the Saffir-Simpson Hurricane Wind Scale), so we can raise the level of forecasting for uncertain demands by using such data. It follows that we need to consider uncertain covariates and uncertain demands to construct a scenario-wise ambiguity set. For example, when an earthquake reaches magnitude VII, it will cause more damage than an earthquake of magnitude V and the demand for relief supplies will also be higher, so earthquake magnitude can be used as a covariate. For simplicity, we further divide the space of covariates into $|\mathcal{S}|$ non-overlapping regions to form $|\mathcal{S}|$ scenarios. This gives rise to the outcome space $\Omega := \{(\tilde{\mathbf{d}}, \tilde{s}) \in \mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|\}$. Let s represent a scenario taking values in $\mathcal{S}$ and p_s denote the probability of scenario s happening. Furthermore, we have $\mathbb{P}(\tilde{s} = s) = p_s$ and $\sum_{s\in\mathcal{S}} p_s = 1$, where $\tilde{s}$ represents a set of random scenarios whose realization probabilities may be uncertain. The joint distribution of $(\tilde{\mathbf{d}}, \tilde{s})$ is denoted by $\mathbb{P} \in \mathcal{F}$. Now, we specify the scenario-wise ambiguity set $\mathcal{F}$ as

$$\mathcal{F} := \left\{ \mathbb{P} \in \mathscr{P}_0(\mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|) : \begin{array}{ll} (\tilde{\mathbf{d}}, \tilde{s}) \sim \mathbb{P} & \\ \mathbb{P}(\tilde{\mathbf{d}} \in \mathcal{D}^s | \tilde{s} = s) = 1, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\mathbf{d}} | \tilde{s} = s] = \boldsymbol{\mu}^s, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{P}}[|\tilde{\mathbf{d}} - \boldsymbol{\mu}^s| | \tilde{s} = s] \leq \boldsymbol{\nu}^s, & s \in \mathcal{S} \\ \mathbb{P}(\tilde{s} = s) = p_s, & s \in \mathcal{S} \end{array} \right\},$$

where $\mathcal{D}^s$ is the support set defined as

$$\mathcal{D}^s := \left\{ \tilde{\mathbf{d}} \in \mathbb{R}^{|\mathcal{L}|} : \underline{\mathbf{d}}^s \leq \tilde{\mathbf{d}} \leq \overline{\mathbf{d}}^s \right\}.$$

In $\mathcal{F}$, it is natural to incorporate the mean, the mean absolute deviation and the support set of the random variable $\tilde{\mathbf{d}}$. Specifically, for different scenarios, the bounded support set defined in the first set of equality constraints may differ. Conditioning on the scenario realization, the mean of $\tilde{\mathbf{d}}$ is specified in the second set of equality constraints, while the third set of inequality constraints provides upper bounds on the mean absolute deviation of $\tilde{\mathbf{d}}$. The last set of equality constraints specifies the probability of each scenario. The scenario-wise ambiguity set requires simple descriptive statistics from data and allows us to model a rich variety of structural information about the uncertain demand for relief supplies.

4.2.2 Model Reformulation.

Note that the scenario-wise ambiguity set $\mathcal{F}$ involves nonlinear moment constraints due to $\mathbb{E}_{\mathbb{P}}[|\tilde{\mathbf{d}} - \boldsymbol{\mu}^s| | \tilde{s} = s] \leq \boldsymbol{\nu}^s$. Following standard techniques in robust optimization (see, for instance, Wiesemann et al., 2014), we introduce an auxiliary probability space $\Omega' := \{(\tilde{\mathbf{d}}, \tilde{\mathbf{z}}, \tilde{s}) \in \mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|\}$ to reformulate the ambiguity set $\mathcal{F}$ as the projection of a lifted ambiguity set $\mathcal{G}$:

$$\mathcal{G} := \left\{ \mathbb{Q} \in \mathscr{P}_0(\mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \times |\mathcal{S}|) : \begin{array}{ll} (\tilde{\mathbf{d}}, \tilde{\mathbf{z}}, \tilde{s}) \sim \mathbb{Q} & \\ \mathbb{Q}((\tilde{\mathbf{d}}, \tilde{\mathbf{z}}) \in \bar{\mathcal{D}}^s | \tilde{s} = s) = 1, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{Q}}[\tilde{\mathbf{d}} | \tilde{s} = s] = \boldsymbol{\mu}^s, & s \in \mathcal{S} \\ \mathbb{E}_{\mathbb{Q}}[\tilde{\mathbf{z}} | \tilde{s} = s] \leq \boldsymbol{\nu}^s, & s \in \mathcal{S} \\ \mathbb{Q}(\tilde{s} = s) = p_s, & s \in \mathcal{S} \end{array} \right\},$$

where $\bar{\mathcal{D}}_s$ is the lifted support set defined as

$$\bar{\mathcal{D}}^s := \left\{ (\tilde{\mathbf{d}}, \tilde{\mathbf{z}}) \in (\mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|}) : \begin{array}{l} \underline{\mathbf{d}}^s \leq \tilde{\mathbf{d}} \leq \overline{\mathbf{d}}^s \\ |\tilde{\mathbf{d}} - \boldsymbol{\mu}^s| \leq \tilde{\mathbf{z}} \end{array} \right\}.$$

Compared with the original ambiguity set $\mathcal{F}$, the lifted ambiguity set $\mathcal{G}$ is a set of distributions of random triplet $(\tilde{\mathbf{d}}, \tilde{\mathbf{z}}, \tilde{s})$. Furthermore, following some recent results in DRO (see, for instance, Bertsimas et al., 2019), we can define the ambiguity set $\mathcal{F}$ as the set of marginal distributions over $(\tilde{\mathbf{d}}, \tilde{s})$ for all $\mathbb{Q} \in \mathcal{G}$. That is, $\mathcal{F} = \prod_{\tilde{\mathbf{d}}, \tilde{s}} \mathcal{G}$. With the lifted ambiguity set $\mathcal{G}$ in hand, we transform problem $\mathrm{P_{DRO}}$ in Lemma 2.

Lemma 2. *The distributionally robust optimization problem P_{DRO} with $\mathcal{F}$, i.e., $\mathcal{F} = \prod_{\tilde{\mathbf{d}}, \tilde{s}} \mathcal{G}$, is equivalent to the following adjustable robust optimization (ARO) problem:*

$$(P_{ARO}) \quad v_n^* :=$$

$$\inf_{\substack{\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{\eta}, \alpha, \{\boldsymbol{y}^s(\cdot)\}_{s \in \mathcal{S}} \\ \boldsymbol{\pi}^1, \boldsymbol{\pi}^2, \boldsymbol{\pi}^3 \geq \mathbf{0}}} \alpha \tag{9a}$$

$$\text{s.t.}\quad \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + \left(\boldsymbol{\pi}^2_{si}\right)'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \le \tau_i, \qquad i \in \mathcal{L}\setminus\bar{\mathcal{L}}^n, \tag{9b}$$

$$\eta_i + \frac{1}{\underline{\alpha}^n_i}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \le \tau_i, \qquad i \in \bar{\mathcal{L}}^n, \tag{9c}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \ge 0, \qquad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, i \in \mathcal{L}, \tag{9d}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \ge -p_s\eta_i, \qquad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, i \in \mathcal{L}, \tag{9e}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \ge p_s(d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d},\mathbf{z})) - \eta_i), \qquad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, i \in \mathcal{L}, \tag{9f}$$

$$\sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) \le r_i, \qquad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, i \in \mathcal{L}, \tag{9g}$$

$$y^s_{ij}(\mathbf{d},\mathbf{z}) \ge 0, \qquad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s, s\in\mathcal{S}, i,j \in \mathcal{L} \tag{9h}$$

$$(1a) - (1d), \alpha \in (0,1], \boldsymbol{\eta} \ge \mathbf{0}. \tag{9i}$$

where each $y^s_{ij} : \mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \to \mathbb{R}$, *and where* π^1_{si}, $\boldsymbol{\pi}^2_{si}$ *and* $\boldsymbol{\pi}^3_{si}$ *are the dual variables associated with first, second, and third constraints that define* $\mathcal{G}$*.*

Proof. Please see Appendix B.4.

Note that problem P_{ARO} is a semi-infinite programming problem with an infinite number of constraints and adaptive decision variables. Specifically, the adaptive decisions y^s_{ij} can be seen as general functions of random vectors $(\tilde{\mathbf{d}}, \tilde{\mathbf{z}})$. Therefore, problem P_{ARO} is not yet directly solvable.

4.3 Identifying optimal adaptable resource reallocation

4.3.1 Exact solution.

Based on a vertex enumeration (VE) method, we first present an exact linear programming reformulation for problem P_{ARO} in Proposition 2.

Proposition 2. *The adjustable robust optimization problem* P_{ARO} *is equivalent to the following mixed-integer linear program:*

$$(P_{VE})\ v^*_n := \inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}\\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\ge\mathbf{0}}} \alpha \tag{10a}$$

$$\text{s.t.}\quad \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \le \tau_i, \qquad i \in \mathcal{L}\setminus\bar{\mathcal{L}}^n, \tag{10b}$$

$$\eta_i + \frac{1}{\underline{\alpha}^n_i}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \le \tau_i, \qquad i \in \bar{\mathcal{L}}^n, \tag{10c}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d}(\omega) + (\boldsymbol{\pi}^3_{si})'\mathbf{z}(\omega) \ge 0, \qquad \omega \in \Omega^s, s\in\mathcal{S}, i \in \mathcal{L}, \tag{10d}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d}(\omega) + (\boldsymbol{\pi}^3_{si})'\mathbf{z}(\omega) \ge -p_s\eta_i, \qquad \omega \in \Omega^s, s\in\mathcal{S}, i \in \mathcal{L}, \tag{10e}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d}(\omega) + (\boldsymbol{\pi}^3_{si})'\mathbf{z}(\omega) \ge p_s(d_i(\omega) + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\omega)$$

$$
\begin{aligned}
&\quad - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\omega)) - \eta_i), && \omega \in \Omega^s, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(10f)}\\
&\sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\omega) \le r_i, && \omega \in \Omega^s, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(10g)}\\
&y^s_{ij}(\omega) \ge 0, && \omega \in \Omega^s, s \in \mathcal{S}, i, j \in \mathcal{L}, && \text{(10h)}\\
&\text{(1a)} - \text{(1d)}, \alpha \in (0,1], \boldsymbol{\eta} \ge \mathbf{0}. && && \text{(10i)}
\end{aligned}
$$

where each $y^s_{ij} : \Omega_s \to \mathbb{R}$, and where, for all $s \in \mathcal{S}$, the set $\{\mathbf{d}(\omega)\}\}_{\omega\in\Omega^s}$ contains all vertices of the bounded polyhedron $\bar{\mathcal{D}}^s_b$ defined as:

$$
\bar{\mathcal{D}}^s_b := \left\{(\tilde{\mathbf{d}}, \tilde{\mathbf{z}}) \in \mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} : \underline{\mathbf{d}}^s \le \tilde{\mathbf{d}} \le \overline{\mathbf{d}}^s,\ |\tilde{d}_i - \mu^s_i| \le \tilde{z}_i \le \max\{\overline{d}^s_i - \mu^s_i, \mu^s_i - \underline{d}^s_i\}\ \forall i \in \mathcal{L}\right\}.
$$

Proof. Please see Appendix B.5.

Note that the number of vertices indexed by Ω^s for all $s \in \mathcal{S}$ grows exponentially with the number of locations. In practice, we can employ a column-and-constraint generation (C&CG) method (see, for instance, Zeng and Zhao, 2013) to speed up the resolution of problem P_{VE}. We however consider further investigation of acceleration schemes to fall beyond the scope of this paper, and instead derive in the next subsection a conservative approximation that takes the form of a more reasonably sized mixed-integer linear programs (MILP).

4.3.2 Affinely Adjustable Robust Counterpart.

As an alternative, we apply the idea of an affine decision rule to address the adaptive decision and solve an approximate problem of $\mathrm{P}_{\mathrm{ARO}}$. More specifically, for each scenario $s \in \mathcal{S}$, we approximate the adaptive decision by an affine function $\boldsymbol{y}^s(\cdot) \in \mathscr{A}$, where

$$
\mathscr{A} := \left\{ \boldsymbol{y} : \mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \to \mathbb{R}^{|\mathcal{L}|\times|\mathcal{L}|} : \begin{array}{l} \exists \boldsymbol{y}^0, \boldsymbol{y}^1_l, \boldsymbol{y}^2_l \in \mathbb{R}^{|\mathcal{L}|\times|\mathcal{L}|}, \forall l \in \mathcal{L} \\ \boldsymbol{y}(\mathbf{d}, \mathbf{z}) = \boldsymbol{y}^0 + \sum_{l\in\mathcal{L}} \boldsymbol{y}^1_l d_l + \sum_{l\in\mathcal{L}} \boldsymbol{y}^2_l z_l \end{array} \right\}.
$$

Based on the affine decision rule, we have the following affinely adjustable robust counterpart (AARC) of problem $\mathrm{P}_{\mathrm{ARO}}$:

$$
\begin{aligned}
&(\mathrm{P}_{\mathrm{AARC}})\ v^{\mathrm{AARC}}_n := \\
&\inf_{\substack{\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{\eta}, \alpha, \{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}} \\ \boldsymbol{\pi}^1, \boldsymbol{\pi}^2, \boldsymbol{\pi}^3 \ge \mathbf{0}}} \alpha && && \text{(11a)}\\
&\text{s.t. } \eta_i + \frac{1}{\alpha} \sum_{s\in\mathcal{S}} \left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \le \tau_i, && i \in \mathcal{L} \setminus \bar{\mathcal{L}}^n, && \text{(11b)}\\
&\eta_i + \frac{1}{\underline{\alpha}^n_i} \sum_{s\in\mathcal{S}} \left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \le \tau_i, && i \in \bar{\mathcal{L}}^n, && \text{(11c)}\\
&\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \ge 0, && (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}^s, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(11d)}\\
&\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \ge -p_s\eta_i, && (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}^s, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(11e)}\\
&\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \ge p_s\big(d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d}, \mathbf{z}) \\
&\quad - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d}, \mathbf{z})) - \eta_i\big), && (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}^s, s \in \mathcal{S}, i \in \mathcal{L}, && \text{(11f)}
\end{aligned}
$$

$$\sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d},\mathbf{z}) \le r_i, \qquad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i\in\mathcal{L}, \tag{11g}$$

$$y_{ij}^s(\mathbf{d},\mathbf{z}) \ge 0, \qquad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, i,j\in\mathcal{L}, \tag{11h}$$

$$\boldsymbol{y}^s(\cdot)\in\mathscr{A}, \qquad s\in\mathcal{S} \tag{11i}$$

$$(1a)-(1d), \alpha\in(0,1], \boldsymbol{\eta}\ge\mathbf{0}. \tag{11j}$$

Since problem P_{AARC} has an infinite number of constraints, it cannot be solved directly. One can alternatively view some constraints in problem P_{AARC} as robust counterparts of the linear optimization problem under the lifted support set $\bar{\mathcal{D}}_s$ for all $s\in\mathcal{S}$. Hence, we can transform it into a linear optimization problem via standard techniques from the robust literature. For presentation brevity, the derivation of the resulting MILP is relegated to Appendix B.6.

Moreover, AARC can also lead to improvements in computational performance. Empirically, AARC appears surprisingly accurate with a maximum measured optimality gap of 8% (shown in Appendix D.3), while significantly reducing the computation time compared to problem P_{VE} and the sample average approximation (SAA) model.Therefore, AARC is able to improve both the quality and the speed of the solution relative to SAA and P_{VE} respectively.

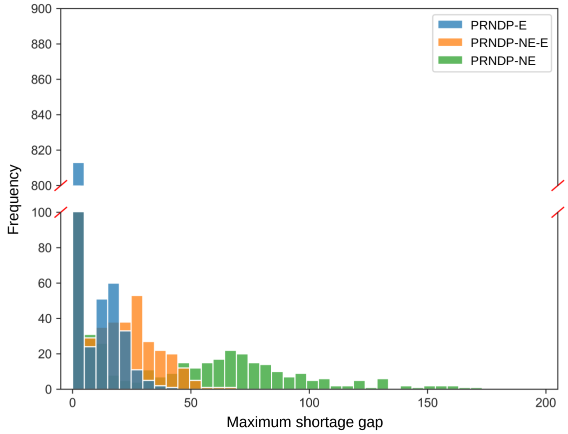
5 Computational Results

In this section, we first compare the PRNDP-E with other models in terms of equity and shortage. Next, we analyze the impact of donations. Finally, we illustrate the benefits of utilizing the scenario-wise ambiguity set. We conduct a series of numerical studies based on a real earthquake case that occurred in Yushu County, Qinghai Province, China in 2010. The deterministic parameters (e.g., cost, capacity, donations, etc.) and demand-related parameters are from Ni et al. (2018), as well as HOs websites. For presentation brevity, most details of the numerical studies are pushed to Appendix D.1. All experiments are carried out on a PC with a 3.6-GHz processor and 16 GB RAM. The models are coded in JAVA and solved using IBM ILOG CPLEX Optimization Studio 12.7.1.

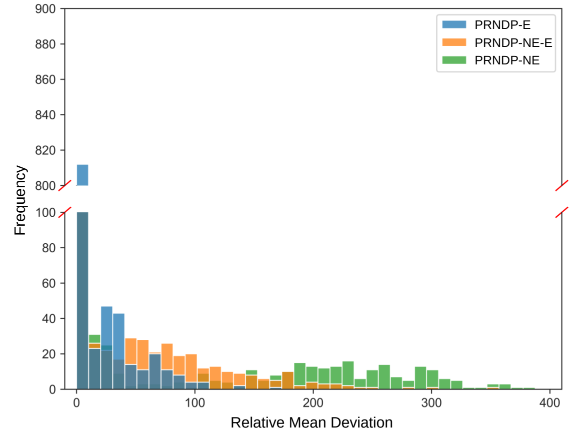
5.1 Impact of equity

We denote the model that considers equity in both predisaster deployment and postdisaster reallocation by PRNDP-E and use PRNDP-NE to refer to the benchmark model without equity in both stages. The detailed formulation of the PRNDP-NE is shown in Appendix C, which minimizes the total shortage. We also investigate the setting where the decision maker considers equity only when reallocating supplies after the disaster (with predisaster deployment fixed to solutoin of PRNDP-NE), denoted by PRNDP-NE-E. For fixed budgets and donations (we vary the fixed budgets and donations to illustrate their impact in Section 5.3), we test three models—PRNDP-E, PRNDP-NE-E, and PRNDP-NE—and compare their out-of-sample performances under the five commonly used equity indices. Specifically, we first solve all the above models to optimality and obtain the optimal predisaster deployment decisions $(\boldsymbol{x}^*, \boldsymbol{r}^*)$. After that, given each of the solutions $(\boldsymbol{x}^*, \boldsymbol{r}^*)$ and an observed sample $(s, \mathbf{d})$ pair, for PRNDP-E and PRNDP-NE-E we solve a deterministic lexicographic optimization problem to minimize the shortage of each location, while for PRNDP-NE we minimize the total shortage. We then examine (i) the Maximum Shortage Gap (MSG); (ii) the Relative Mean Deviation (RMD); (iii) the Variance (VAR); (iv) the Sum of Pairwise Absolute Differences (SPAD); and (v) the Gini coefficient (Gini) of 1,000 test (out-of-sample) samples. Note that these five indices are described in Karsu and Morton (2015) and in a discussion regarding equity in OR/MS areas; their definitions can be found in Appendix D.2.

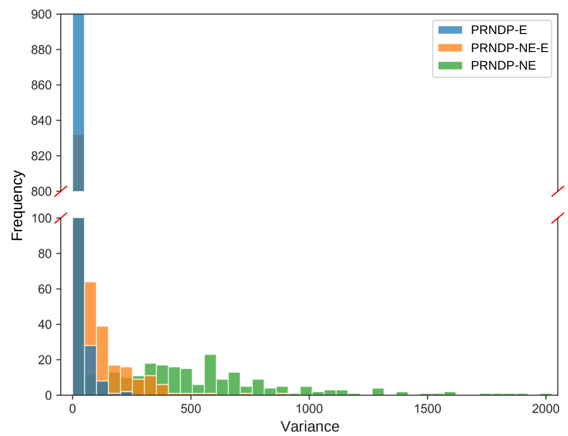
Figure 2 presents the distributions of MSG, RMD, VAR and SPAD values for 1,000 test samples under the PRNDP-E, the PRNDP-EE, and the PRNDP-NE solutions. First, note that, for all tested equity indices,



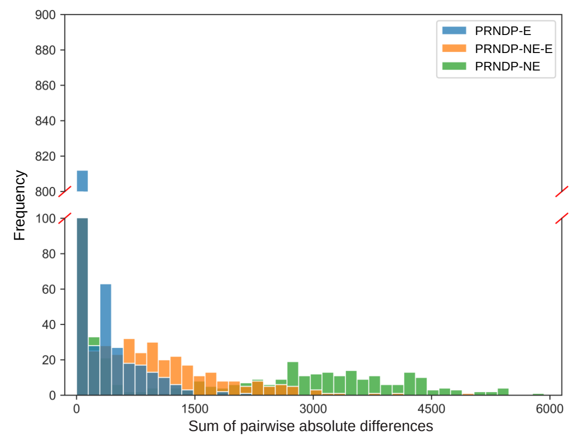
(a) MSG



(b) RMD



(c) VAR



(d) SPAD

Figure 2: Frequency distribution of equity indices of out-of-sample data

the PRNDP-E and the PRNDP-NE-E perform better than the PRNDP-NE because a smaller index value is more desirable. It indicates the importance of incorporating equity in the allocation of relief resources. Second, we can observe that the performance of PRNDP-E is better than PRNDP-NE-E. The result shows that compared with considering equity in reallocation after a disaster, incorporating equity already in the stage of predisaster deployment can further improve the equity of resource allocation. This confirms that HOs can draw real value from jointly considering deployment and reallocation strategies when searching for the most equitable allocation of relief supplies. More precisely, when it comes to the mean, the 95%VaR, the 99%VaR, and the standard deviations (STD), the PRNDP-E approach also outperformed the other two other approaches (see Table 2 for detailed statistics).

We further conduct experiments on the Gini coefficient. Since the Gini coefficient is often represented graphically through the Lorenz curve, Figure 3 shows the Lorenz curve of shortage distributions by plotting the location percentile by shortages on the x-axis and cumulative shortages on the y-axis. We can observe that the curve of PRNDP-E is closer to the line of perfect equality with a Gini coefficient equal to 0.08, which is smaller than the results of the other two approaches.

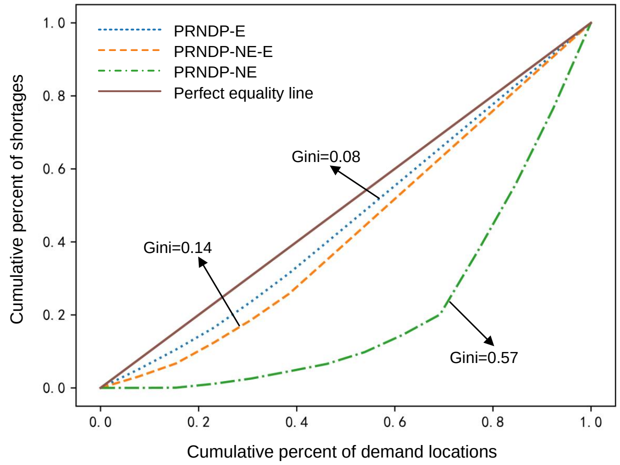


Figure 3: Gini coefficient of out-of-sample data under PRNDP-E, PRNDP-NE-E, and PRNDP-NE

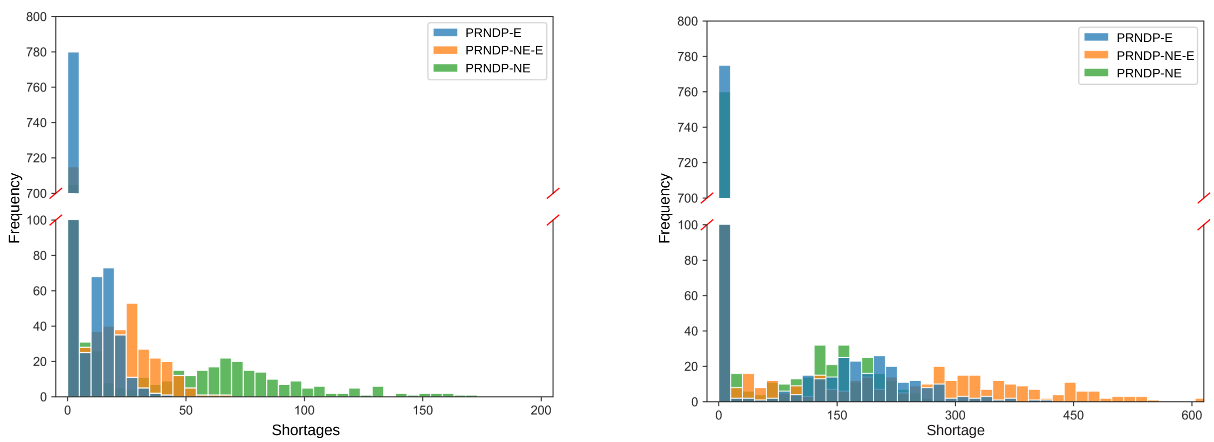
To capture more of the impact of equity, we conduct more numerical experiments. Specifically, we first add a perturbation Δ to the mean of the demand of out-of-sample data, that is $\boldsymbol{\mu}^s_{out} := (1+\Delta)\boldsymbol{\mu}^s$ for all $s \in \mathcal{S}$. Then we generate 1,000 test samples from the out-of-sample distribution according to the value of Δ (see Appendix D.1 for details). In Table 2, we summarize the out-of-sample performance of equity indices under the PRNDP-E, the PRNDP-NE-E, and the PRNDP-NE. We observe that on the average and in the extremes (VaR@95% and VaR@99%) the PRNDP-E continues to outperform the other two approaches with regard to all equity indices. This observation suggests that the PRNDP-E approach does help address equity concerns in humanitarian operations.

Table 2: Out-of-sample statistics of equity index under PRNDP-E, PRNDP-NE-E and PRNDP-NE

		PRNDP-E				PRNDP-NE-E				PRNDP-NE			
delta	Statistic	MSG	RMD	VAR	SPAD	MSG	RMD	VAR	SPAD	MSG	RMD	VAR	SPAD
-0.10	Mean	0.44	1.42	0.44	18.51	2.64	6.23	5.22	96.74	3.57	7.97	6.98	107.58
	Var@95%	3.02	9.15	1.15	118.92	18.04	49.43	43.44	633.80	25.26	54.24	49.04	740.64
	Var@99%	10.66	36.08	14.79	469.03	28.74	53.43	107.43	1,034.60	37.69	63.08	118.37	1,251.48
	STD	1.80	6.18	32.81	80.38	6.50	18.85	280.38	178.14	8.76	19.96	306.21	271.43
-0.05	Mean	1.69	4.89	2.73	63.71	4.84	19.07	19.16	250.35	8.50	23.22	35.43	322.19
	Var@95%	11.99	36.85	16.37	479.00	28.61	121.38	133.17	1,596.64	49.46	137.75	208.64	1,941.40
	Var@99%	19.56	73.78	53.46	959.19	41.29	181.62	277.25	2,371.20	74.52	214.92	537.53	3,042.16
	STD	4.49	14.48	144.08	188.36	10.05	41.90	727.23	551.33	17.83	50.66	1,261.12	709.21
0.00	Mean	3.23	8.53	6.51	111.60	7.25	25.94	31.01	341.42	17.59	51.02	130.17	727.48
	Var@95%	20.00	58.38	39.26	758.99	37.05	138.79	196.94	1,804.22	90.37	277.77	732.13	3,976.16
	Var@99%	28.03	94.89	102.31	1,233.51	47.49	207.78	359.79	2,701.20	142.62	339.63	1,508.01	5,086.36
	STD	7.17	21.09	270.41	276.09	13.13	50.49	1,049.24	667.98	33.93	97.39	3,992.42	1,400.84
0.05	Mean	4.45	11.27	11.02	148.03	9.44	31.59	43.20	415.80	25.64	76.53	267.16	1,104.63
	Var@95%	28.61	75.43	76.52	985.44	45.27	154.51	266.72	2,047.20	136.54	392.51	1,607.64	5,718.00
	Var@99%	36.73	115.09	151.12	1,500.59	58.47	231.00	534.87	3,090.96	163.06	468.51	2,148.72	6,811.64
	STD	9.71	26.51	419.41	347.61	15.94	55.96	1,338.42	737.92	46.92	141.67	7,390.43	2,058.87
0.10	Mean	5.70	14.14	15.50	185.58	11.57	35.18	50.97	464.15	33.54	104.70	446.90	1,526.94
	Var@95%	34.65	79.45	98.29	1,081.31	49.02	157.20	268.66	2,055.68	163.81	520.96	2,616.08	7,610.88
	Var@99%	43.10	135.41	206.89	1,760.28	62.97	213.50	517.99	2,799.72	182.17	587.16	3,096.05	8,519.88
	STD	11.65	30.45	575.97	399.74	18.13	56.36	1,443.08	743.67	58.29	187.71	11,796.93	2,761.86

5.2 Impact of shortages

To evaluate the impact of shortages under the PRNDP-E, the PRNDP-NE-E, and the PRNDP-NE, we compare in Figure 4 the shortage performance for out-of-sample data. Specifically, Figure 4(a) shows the shortage performance of the participants with the worst experience. We find that the PRNDP-E approach is superior to the other two approaches. This observation suggests that we can use the PRNDP-E approach to support the most vulnerable groups regardless of where they are. For total shortage performance, Figure 4(b) shows the frequency distribution of the total shortage for out-of-sample data. We observe that the total shortage derived from the PRNDP-E approach is very close to the PRNDP-NE which minimizes the total shortage. On the other hand, although the PRNDP-NE-E can reduce the shortage experienced by the worst participants, the performance of the total shortage caused by it is much worse than that of the PRNDP-E. Our results show that (i) incorporating equity in both predisaster deployment and postdisaster reallocation can help alleviate shortages among the most vulnerable groups without worsening the overall shortage performance; (ii) incorporating equity only in the postdisaster response can indeed reduce the shortage of the worst participants, but comes at the cost of significantly increasing the total shortage.



(a) Worst

(b) Sum

Figure 4: Distribution of shortage performance of out-of-sample data. (a) presents the distribution of worst shortage among all locations, while (b) presents the distribution of the sum of shortages.

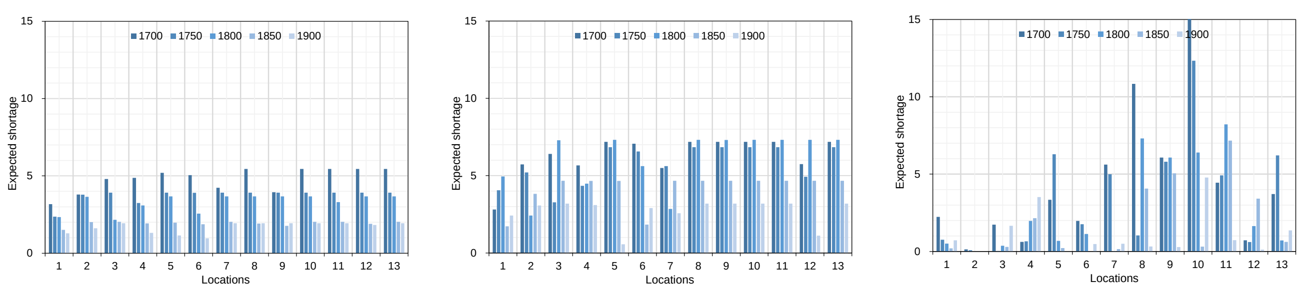
Similarly, we evaluate more instances with different Δ. Table 3 provides a summary of shortage performance for out-of-sample data. We note that compared with the PRNDP-NE, the PRNDP-E can significantly reduce the shortage of the worst participants and only slightly increase the total shortage (because the PRNDP-NE minimizes the total shortage), while the PRNDP-NE-E will make the total shortage performance very poor. Our experiments suggest that HOs can use the PRNDP-E to support the most vulnerable groups while maintaining a low total shortage.

Table 3: Out-of-sample statistics of shortage under PRNDP-E, PRNDP-NE-E, and PRNDP-NE

		PRNDP-E		PRNDP-NE-E		PRNDP-NE	
Delta	Statistic	Worst	Sum	Worst	Sum	Worst	Sum
-0.1	Mean	0.46	5.20	2.64	27.10	3.57	4.70
	Var@95%	3.41	38.06	18.04	190.60	25.26	31.25
	Var@99%	10.66	117.26	28.74	290.07	37.69	57.12
	STD	1.81	20.00	6.50	65.88	8.76	12.00
-0.05	Mean	1.84	21.03	4.85	50.97	8.50	14.94
	Var@95%	12.36	143.24	28.61	296.95	49.46	92.25
	Var@99%	19.56	221.19	41.29	421.58	74.52	148.82
	STD	4.58	51.60	10.04	104.30	17.83	33.55
0	Mean	3.68	42.83	7.31	78.78	17.59	35.03
	Var@95%	20.04	234.48	37.05	392.10	90.37	192.89
	Var@99%	28.03	311.33	47.49	500.50	142.62	238.45
	STD	7.45	85.74	13.14	140.31	33.93	68.08
0.05	Mean	5.28	62.17	9.50	104.03	25.64	53.89
	Var@95%	28.65	327.09	45.27	493.90	136.54	279.80
	Var@99%	36.73	411.40	58.47	604.14	163.06	327.09
	STD	10.29	120.37	15.95	174.63	46.92	101.43
0.1	Mean	7.22	85.67	11.64	129.96	33.54	76.03
	Var@95%	35.26	426.86	49.02	549.36	163.81	385.02
	Var@99%	43.10	487.46	62.97	697.44	182.17	438.39
	STD	12.95	153.73	18.15	204.44	58.29	139.53

5.3 Impact of Donations and Budget Constraints

Donations and budgets are major concerns of the HOs in predisaster deployment; they also directly affect the shortage situation after the disaster occurs. If the HOs do not receive adequate donations and funds for the prepositioning of emergency supplies, the vulnerable population will inevitably face a relief shortage if a disaster occurs. Since donations and budgets have similar effects, we focus on the impact of different donation levels on shortages. Specifically, we set the total amount of supplies to $R = \{1700, 1750, 1800, 1850, 1900\}$ and evaluate the impact of different donations.



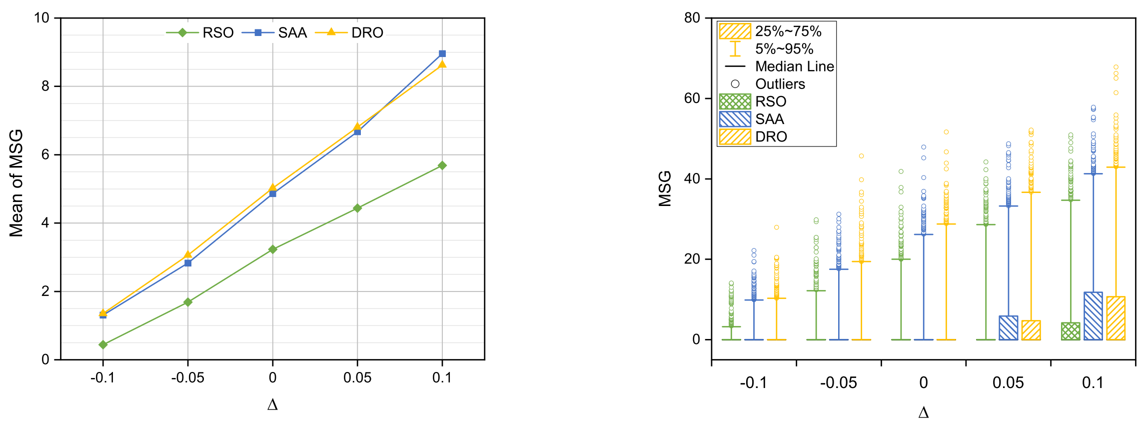
(a) PRNDP-E (b) PRNDP-NE-E (c) PRNDP-NE

Figure 5: Expected shortage at each location under different donation levels of out-of-sample data

Figure 5 reports the out-of-sample expected shortage at each location under different donation levels. First, in Figure 5(a), the expected shortage at each location shows a decreasing trend as donations increase, which seems to be in line with HOs' expectations. However, if equity is not fully considered, then we find that increasing donation does not necessarily reduce shortages and even increase shortages in some locations. For example, as shown in Figure 5(c), when the total amount of supplies increase from 1,750 to 1,800, the shortage of location 11 increase instead. It indicates that HOs cannot simply increase the amounts of supplies to reduce the shortages at each location, but can do so through equitable and reasonable resource allocation. As stated by Besiou and Van Wassenhove (2020): "Reduced funding calls for careful prioritization and cost-effectiveness, but it should be noted that this may conflict with equity and other ethical considerations." Our work can give an equitable solution when the budget is reduced. Second, we can find that the expected shortage reduction is not symmetric across all locations and not necessarily proportional to the percentage increase in donations, which is consistent with Observation 6 in Stauffer and Kumar (2021) for the predisaster deployments. Third, the shortage gaps between demand locations under PRNDP-E is smaller than under PRNDP-NE-E and PRNDP-NE, which demonstrates the equity efficiency of PRNDP-E from another perspective.

5.4 Impact of Information

The RSO framework unifies both scenario-tree-based stochastic optimization (SAA) and distributionally robust optimization (DRO). To further illustrate the performance of RSO model, we compare the out-of-sample performance of the three models in dealing with equity concerns. Specifically, in Figure 6, we report the Maximum Shortage Gap (MSG) measured on the out-of-sample data. Figure 6(a) presents the average of out-of-sample MSG under the RSO, DRO and SAA models for 1,000 test samples. We observe that the RSO model delivers a significantly better performance than the DRO and SAA models. In particular, when the out-of-sample demands are higher (i.e., Δ increases from -0.1 to 0.1), the gap between the RSO and the other two models becomes greater. This confirms that the scenario-wise ambiguity set helps to more clearly describe the uncertainty of relief demand and alleviate the inequities caused by uncertain shortages.



(a) (b)

Figure 6: Out-of-Sample performance of RSO, DRO, and SAA models in Earthquake case

Moreover, to compare the stability of the models for out-of-sample data, we report the box plot of out-of-sample MSG under RSO, DRO and SAA models in Figure 6(b). We find that the values under the RSO model have the smallest variation between both $(25\% - 50\%)$ and $(5\% - 95\%)$, while DRO and SAA incur a relatively large variation. Furthermore, we notice that even for outliers exceeding 95%, the performance of RSO is better than that of DRO and SAA. In general, the out-of-sample data adaptability of the RSO model is higher than that of the DRO and SAA models. This is because the RSO model incorporates empirical scenario information to make the ambiguity set better capture the statistics of the realization, which is useful for improving the efficiency of disaster management.

6 Managerial Insights and Conclusions

Our work considers three important practical aspects of humanitarian operations: shortages, equity, and uncertainty. Mathematically, we propose a new branch-and-bound algorithm for the mixed-integer lexicographic optimization problem with non-convex objectives and prove its optimality. To identify optimal adaptable resource reallocation, we propose two approaches: an exact approach and a conservative approximation that allows us to solve instances of realistic size.

Our research also proposes several managerial recommendations for the HOs:

- There is no conflict between ensuring that participants or individuals have equitable access to relief resources and reducing the total shortage. The goal of HOs is usually to minimize the total shortage while ignoring the plight of individual beneficiaries. However, if one of the participants faces a serious shortage, the media will focus on it and criticize HOs for the inequity in their allocation of resources. Numerical experiments show that HOs can respond to the needs of the world's most vulnerable people by incorporating equity into the predisaster deployment and postdisaster reallocation. Specifically, the results in Sections 5.1 and 5.2 have demonstrated the capability of our approach in improving the performances of the equity indices and reducing shortage.

- While donation and budget constraints limit the amount of relief resources available to each beneficiary, the individual shortage may not be alleviated with increased budgets and donations if equity is not fully incorporated. To respond to the needs of the most vulnerable people, HOs often request increased relief assistance. This does help reduce the total shortage, but that is not necessarily the case for individuals. Specifically, the results in Section 5.3 show that the shortages experienced by some participants do not decrease proportionally or even increase with an increase in donations. By incorporating equity into both the predisaster deployment and postdisaster reallocation, the shortages of all beneficiaries show a downward trend as donations increase, which meets the expectations of HOs.

- Disaster scenario-wise information, if properly segmented, can help alleviate inequities caused by uncertain relief demands. While disasters are extremely unpredictable and relief demands are difficult to accurately estimate, the use of historical data and/or prior knowledge can improve decision-making effects. Our numerical studies in Section 5.4 demonstrate that the scenario-wise ambiguity sets outperform the classical moment ambiguity sets and sample average approximation in alleviating inequities. HO practitioners can construct scenario-wise ambiguity sets based on historical data, which may be regarded as a useful exploration of data-driven methods in disaster relief management.

This research focuses on single-period resource reallocation in postdisaster response. One possible future research direction is to consider the multi-period resource reallocation problem, which may go beyond the research scope of predisaster network design and belongs to the postdisaster response stage. In addition, this research focuses on the uncertainties in relief demands. Thus, a potential future research direction is to incorporate the uncertainties in supplies and in road capacities.

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Appendix

The appendix is organized as follows. Appendix A shows the motivation to develop a new branch-and-bound algorithm. Appendix B provides the proofs for all results in the paper. Appendix C shows the formulation of PRNDP-NE. Appendix D presents all data and setup for numerical studies.

A Motivation for designing the branch-and-bound algorithm

We show the motivation to develop the branch-and-bound algorithm. Since problem P_{LM} can be regarded as a lexicographic minimization problem, a natural idea is whether the state-of-the-art algorithm proposed by Qi (2017) can be directly applied to problem P_{LM}. For convenience, we refer to the algorithm proposed by Qi (2017) as LMP (Lexicographic Minimization Procedure). In the following, we give two examples to illustrate that although the LMP can perform well on linear programming problems, it cannot guarantee optimality for mixed-integer programming problems and non-convex problems.

Example 1 Consider the following feasibility problem :

$$
\begin{aligned}
&2x \le \alpha_1,\\
&2 - x \le \alpha_2,\\
&x \in \{0, 1\},
\end{aligned}
$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]$ is a vector. In this example, we want to find a vector $\boldsymbol{\alpha}$ that is lexicographically minimal. By the LMP, one can get $\boldsymbol{\alpha} = [2, 1]$ with $x = 1$. However, we may have $\boldsymbol{\alpha'} = [0, 2]$ with $x = 0$. Obviously, $\boldsymbol{\alpha'}$ is lexicographically less than $\boldsymbol{\alpha}$. Based on the analysis of these two solutions, we observe that, there are two sets of possible binding constraints that correspond to different solutions for the minimum scalar value $\alpha^* = 2$. Yet, the LMP is unable to decide which binding constraint to fix and instead could arbitrarily fix the first one detected. It follows that the LMP sometimes fails to find the lexicographically minimum vector.

Example 2 Consider an example of problem P_{LM} with $|\mathcal{L}| = 5$ and $|\mathcal{K}| = 1$. Specifically, we will let the parameters $M = 200$, $R = 300$, $B = 400$, $\boldsymbol{c} = \{200, 200, 150, 150, 300\}$, $\boldsymbol{\tau} = \{3, 33, 25, 34, 21\}$. Furthermore, as shown in Table 4, we assume that there are $|\Omega| = 5$ possible outcomes of random demands with the same probability.

Table 4: Random demand for each location in Example 2

		Location				
Outcome	Prob.(%)	1	2	3	4	5
$\omega = 1$	20	40	45	42	48	32
$\omega = 2$	20	9	14	45	28	47
$\omega = 3$	20	45	30	36	92	97
$\omega = 4$	20	85	70	53	13	84
$\omega = 5$	20	54	69	12	74	11

Table 5: Detailed optimal solution under the LMP and the B&B

	LMP			B&B		
Location	$\boldsymbol{x}^*$	$\boldsymbol{r}^*$	$\boldsymbol{\rho}(\boldsymbol{u}^*)$	$\boldsymbol{x}^*$	$\boldsymbol{r}^*$	$\boldsymbol{\rho}(\boldsymbol{u}^*)$
1	1	150	0.00	0	0	0.00
2	1	50	0.28	0	0	0.28
3	0	0	0.00	1	150	0.00
4	0	0	0.28	1	50	0.28
5	0	0	0.28	0	0	0.00

In this example, based on the LMP, one can identify that an optimal solution takes the configuration presented in Table 5. On the other hand, we also show the details of an optimal solution obtained by our branch-and-bound (B&B) procedure in Table 5. Note that the SSM vector derived by the B&B procedure is lexicographically less than that derived by the LMP. The possible reason is that there may be multiple optimal solutions $\boldsymbol{u}_h^*$ at iteration h such that $\max_{j \in \mathcal{L}_h} \rho_{\tau_j}(u_{hj}^*) = \alpha_h^*$ where α_h^* is the minimum scalar value, and LMP randomly chooses one of the optimal solutions to detect an index j such that $\rho_{\tau_j}(u_{hj}^*) = \alpha_h^*$. Different optimal solutions may correspond to different indexes, and different optimization sequences may produce different vectors. Therefore, the LMP sometimes fails to find the lexicographically minimum vector, which motivated us to develop a new algorithm. For the B&B procedure, we refer the reader to Appendix B.3 for the proof of its optimality.

B Proofs

B.1 Proof of Lemma 1.

We can first show that for any $\mathbb{P} \in \mathcal{F}$:

$$\inf_{\eta} \left(\eta + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right) \leq \tau \Leftrightarrow \inf_{0 \leq \eta \leq \tau} \left(\eta + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right) \leq \tau \,.$$

This follows from the fact that any $\eta > \tau$ necessarily leads to $\eta + (1/\alpha)\mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \geq \eta > \tau$. On the other hand, for any $\eta < 0$, we can confirm that $\bar{\eta} := 0$ is such that:

$$\eta + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] = \frac{(\alpha - 1)\eta}{\alpha} + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[\tilde{u} \right] \geq \frac{(\alpha - 1)\bar{\eta}}{\alpha} + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[\tilde{u} \right] = \bar{\eta} + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \bar{\eta})^+ \right] \,,$$

where we used in order the fact that $\tilde{u} \geq 0$, that $(\alpha - 1)/\alpha$ is negative and $\bar{\eta} > \eta$, and again $\tilde{u} \geq 0$.

We can then exploit the representation of CVaR (Rockafellar et al., 2000a) and follow the steps:

$$\begin{aligned} (3) &\Leftrightarrow \forall \mathbb{P} \in \mathcal{F},\, \mathrm{CVaR}_{1-\alpha}^{\mathbb{P}}(\tilde{u}) \leq \tau \Leftrightarrow \forall \mathbb{P} \in \mathcal{F},\, \inf_{\eta} \left(\eta + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right) \leq \tau \\ &\Leftrightarrow \sup_{\mathbb{P} \in \mathcal{F}} \inf_{0 \leq \eta \leq \tau} \left(\eta + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right) \leq \tau \Leftrightarrow \inf_{0 \leq \eta \leq \tau} \sup_{\mathbb{P} \in \mathcal{F}} \left(\eta + \frac{1}{\alpha} \mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] \right) \leq \tau \Leftrightarrow (4), \end{aligned}$$

where the fourth $\Leftrightarrow$ follows from applying Sion's minimax theorem (Sion, 1958) given that η is contained in a compact set, $\mathcal{F}$ is convex and $\eta + (1/\alpha)\mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right]$ is convex in η and affine in $\mathbb{P}$ (Rockafellar et al., 2000b). Finally, the last $\Leftrightarrow$ follows again from the fact that any $\eta > \tau$ necessarily makes $\eta + (1/\alpha)\mathbb{E}_{\mathbb{P}} \left[(\tilde{u} - \eta)^+ \right] > \tau$ for any $\mathbb{P} \in \mathcal{F}$. □

B.2 Proof of Proposition 1.

The proofs of *Monotonicity*, *Satisfaction*, and *Dissatisfaction* are similar to that of Proposition 1 in Qi (2017). For the proof of *Quasi-convexity*, we refer readers to Proposition 3.4 in Mafusalov and Uryasev (2018). □

B.3 Proof of Theorem 1.

We first introduce the notation $n := (i_1, i_2, \ldots, i_k)$, with each $i_j \in \mathcal{L}$ for some $k \in \{1, \ldots, |\mathcal{L}|\}$ to describe the sequence of branching that was traversed to generate node n, which implies that $\bar{\mathcal{L}}^n := \{i_1, \ldots, i_k\}$ and that the descendants of $n = (i_1, i_2, \ldots, i_k)$ are the nodes $n' := (i_1, i_2, \ldots, i_k, j)$ with $j \in \mathcal{L}/\{i_1, \ldots, i_k\}$. Moreover, we will exploit the fact that problem (7) is equivalent to

$$\begin{aligned} &\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\bar{\mathcal{L}}^n} \beta_i \\ &\text{s.t. } \beta_i \leq \underline{\alpha}_i^n,\ i \in \bar{\mathcal{L}}^n, \end{aligned}$$

where $\mathscr{B} := \left\{ \boldsymbol{\beta} \in \mathbb{R}^{|\mathcal{L}|} : \exists \boldsymbol{u} \in \mathscr{U}, \boldsymbol{\beta} = \boldsymbol{\rho}(\boldsymbol{u}) \right\}$.

Lemma 3. *Given any node $n = (i_1, i_2, \ldots, i_k)$ expanded during the resolution of Algorithm 1, we have that $\underline{\vec{\alpha}}_j^n = \underline{\alpha}_{i_j}^n$ for all $j = 1, \ldots, k$ and $\underline{\vec{\alpha}}_j^n = -\infty$ for $j > k$.*

Note that Lemma 3 and the design of Algorithm 1 allows us to use $\vec{\underline{\alpha}}_j^n = \underline{\alpha}_{i_j}^n = \vec{\bar{\alpha}}_j^n = \bar{\alpha}_{i_j}^n$ interchangeably when $n = (i_1, i_2, \ldots, i_k)$ is expanded and $j \le k$.

Proof. This follows from the fact that for $j = 1, \ldots, k-1$, the optimal value $v_{n'}^*$ of problem (7) associated to $n' = (i_1, \ldots, i_{j-1})$ is necessarily greater or equal to the optimal value $v_{n''}^*$ associated to its child node $n'' = (i_1, \ldots, i_j)$:

$$\begin{aligned} \underline{\alpha}_{i_j}^n = &\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{i_1, \ldots, i_{j-1}\}} \beta_i & = &\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{i_1, \ldots, i_{j-1}\}} \beta_i \\ &\text{s.t. } \beta_i \le \underline{\alpha}_i^n,\ i \in \{i_1, \ldots, i_{j-1}\} & &\text{s.t. } \beta_i \le \underline{\alpha}_i^n,\ i \in \{i_1, \ldots, i_j\} \\ \ge &\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{i_1, \ldots, i_j\}} \beta_i & = &\underline{\alpha}_{i_{j+1}}^n \\ &\text{s.t. } \beta_i \le \underline{\alpha}_i^n,\ i \in \{i_1, \ldots, i_j\} \end{aligned}$$

where the second equality follows from the fact that we added the constraint $\beta_{i_j} \le \underline{\alpha}_{i_j}^n$ which is a relaxation of $\max_{i \in \mathcal{L}/\{i_1, \ldots, i_{j-1}\}} \beta_i \le \underline{\alpha}_{i_j}^n$, namely that the objective value is smaller or equal to its optimal value. On the other hand, by construction we have that $\underline{\alpha}_i^n = -\infty$ for all $i \notin \{i_1, \ldots, i_k\}$. □

We now prove Theorem 1 by contradiction. Suppose that there exists a vector $\bar{\boldsymbol{\beta}} \in \mathscr{B}$ such that $\bar{\boldsymbol{\beta}} \prec \boldsymbol{\alpha}^*$; that is, we have $\bar{\boldsymbol{\beta}} \preceq \boldsymbol{\alpha}^*$ but $\boldsymbol{\alpha}^* \not\preceq \bar{\boldsymbol{\beta}}$. Without loss of generality, we can assume that $\vec{\bar{\boldsymbol{\beta}}} = \bar{\boldsymbol{\beta}}$, thus $\bar{\boldsymbol{\beta}} \prec \boldsymbol{\alpha}^*$ implies that there exists a $k \in [1, |\mathcal{L}|]$ such that $\bar{\beta}_i = \vec{\alpha}_i^*$ for all $i < k$ and $\bar{\beta}_k < \vec{\alpha}_k^*$.

If $k = 1$, then $\bar{\beta}_1 < \vec{\alpha}_1^*$ should be true. By Algorithm 1, we have that $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^{n_0}$, so that $\vec{\alpha}_1^* \le \max_i \bar{\alpha}_i^{n_0} = \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i \le \max_i \bar{\beta}_i = \bar{\beta}_1$. This is a contradiction.

If $k > 1$, then $\bar{\beta}_i = \vec{\alpha}_i^*$ for all $i < k$ and $\bar{\beta}_k < \vec{\alpha}_k^*$ should hold. To discuss this case, we will need to make use of the following lemmas.

Lemma 4. *Given any node n expanded by Algorithm 1, it must be that $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^n$.*

Proof. This conclusion can be drawn directly from the definition of Algorithm 1. Specifically, throughout the procedure the vector $\bar{\boldsymbol{\alpha}}$ can only decrease according to the lexicographic ordering. Furthermore, when a node n is expanded, either $\bar{\boldsymbol{\alpha}}^n \succ \bar{\boldsymbol{\alpha}}$ or the latter is replaced with $\bar{\boldsymbol{\alpha}}^n$. □

Lemma 5. *If it is expanded, the node $n^* = (1, \ldots, k-1)$ is such that $\bar{\alpha}_i^{n^*} = \bar{\beta}_i$ for all $i \le k$.*

Proof. We show that $\bar{\alpha}_i^{n^*} = \bar{\beta}_i$ for all $i \le k$ by recursion using $k' \in \{0, \ldots, k\}$. Starting with $k' = 0$, we have that $\bar{\alpha}_1^{n^*} = v_{n_0}^* = \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i = \bar{\beta}_1$, where the first two equalities follows by construction, while the last one follows from $\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i \le \bar{\beta}_1$ since $\bar{\boldsymbol{\beta}} \in \mathscr{B}$, and $\bar{\beta}_1 \le \min_{\boldsymbol{\beta} \in \mathscr{B}} \max_i \beta_i$ since otherwise there exists a $\bar{\boldsymbol{\beta}}' \prec \bar{\boldsymbol{\beta}}$.

We then need to show that if the lemma is true for some $k' < k$, then it is also true for $k'+1$. In particular, letting $n = (1, \ldots, k')$ and $n' = (1, \ldots, k'+1)$, we have by construction that $\bar{\alpha}_i^{n'} = \underline{\alpha}_i^{n'} = \bar{\alpha}_i^n$ for all $i \le k'+1$ since $\bar{\mathcal{L}}^{n'} = \{1, \ldots, k'+1\}$. Yet, we have that $\bar{\alpha}_i^n = \bar{\beta}_i$ for all $i \le k'$. We are left with demonstrating that $\bar{\alpha}_{k'+1}^n = \bar{\beta}_{k'+1}$, which follows from:

$$\begin{aligned} \bar{\alpha}_{k'+1}^n = &\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{1, \ldots, k'\}} \beta_i & = &\min_{\boldsymbol{\beta} \in \mathscr{B}} \max_{i \in \mathcal{L}/\{1, \ldots, k'\}} \beta_i & = \bar{\beta}_{k'+1} \\ &\text{s.t. } \beta_i \le \underline{\alpha}_i^n,\ i \in \{1, \ldots, k'\} & &\text{s.t. } \beta_i \le \bar{\beta}_i,\ i \in \{1, \ldots, k'\} \end{aligned}$$

where we exploited the fact that $\underline{\alpha}_i^n = \bar{\alpha}_i^n = \bar{\beta}_i$ for $i \le k'$, and where the last equality follows from the fact that $\bar{\boldsymbol{\beta}}$ is lexicographic minimal in $\mathscr{B}$. □

Now in the case that node $n^* = (1, \ldots, k)$ is expanded by Algorithm 1, it is clear by Lemma 4 that $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^{n^*}$, which either implies, according to Lemma 5, that $\vec{\alpha}_i^* = \bar{\alpha}_i^{n^*} = \bar{\beta}_i$ for all $i \leq k$ or that there exists a $k' \leq k$ such that $\vec{\alpha}_i^* = \bar{\alpha}_i^{n^*} = \bar{\beta}_i$ for all $i < k'$ while $\vec{\alpha}_{k'}^* < \bar{\alpha}_{k'}^{n^*} = \bar{\beta}_{k'}$. Both situations either lead to a contradiction with respect to the fact that $\bar{\beta}_k < \vec{\alpha}_k^*$ or the fact that $\vec{\alpha}_i^* = \bar{\beta}_i$ for $i < k$, respectively.

We are finally left with the possibility that n^* was not expanded by Algorithm 1. This can only occur if there is a node $\bar{n} = (1, \ldots, \bar{k})$ for some $\bar{k} \leq k$, i.e. among the ancestors, that was not expanded at Step (7). Hence, $\underline{\boldsymbol{\alpha}}^{\bar{n}} \succeq \boldsymbol{\alpha}^*$ and since $\bar{n}$ is an ancestor of n^*, we also have that $\bar{\boldsymbol{\alpha}}^{n^*} \succeq \underline{\boldsymbol{\alpha}}^{\bar{n}}$. Thus $\boldsymbol{\alpha}^* \preceq \bar{\boldsymbol{\alpha}}^{n^*}$, which once again leads to a contradiction. $\square$

B.4 Proof of Lemma 2.

By Definition 1, we first rewrite problem $\mathrm{P_{DRO}}$ using the defined outcome space $(\tilde{\mathbf{d}}, \tilde{s})$ in the scenario-wise ambiguity set $\mathcal{F}$:

$$
\begin{aligned}
\inf_{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}} \quad & \alpha \\
\text{s.t.} \quad & \eta_i + \frac{1}{\alpha} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[f(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}), \tilde{\mathbf{d}}) - \eta_i \right]^+ \leq \tau_i, && i \in \mathcal{L} \setminus \bar{\mathcal{L}}^n, \\
& \eta_i + \frac{1}{\underline{\alpha}_i^n} \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[f(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}), \tilde{\mathbf{d}}) - \eta_i \right]^+ \leq \tau_i, && i \in \bar{\mathcal{L}}^n, \\
& \sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d}) \leq r_i, && \mathbf{d} \in \mathcal{D}^s,\ s \in \mathcal{S},\ i \in \mathcal{L}, \\
& y_{ij}^s(\mathbf{d}) \geq 0, && \mathbf{d} \in \mathcal{D}^s,\ s \in \mathcal{S},\ i, j \in \mathcal{L}, \\
& (1\text{a}) - (1\text{d}), \alpha \in (0, 1], \boldsymbol{\eta} \geq \mathbf{0},
\end{aligned}
$$

where $y_{ij}^s : \mathbb{R}^{|\mathcal{L}|} \to \mathbb{R}_+$ and $\tilde{y}_{ij} := y_{ij}^{\tilde{s}}(\tilde{\mathbf{d}})$. When replacing $\mathcal{F} = \prod_{\tilde{\mathbf{d}},\tilde{s}} \mathcal{G}$, we then obtain:

$$
\begin{aligned}
\inf_{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}} \quad & \alpha && && \text{(12a)} \\
\text{s.t.} \quad & \eta_i + \frac{1}{\alpha} \sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}} \left[f(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}), \tilde{\mathbf{d}}) - \eta_i \right]^+ \leq \tau_i, && i \in \mathcal{L} \setminus \bar{\mathcal{L}}^n, && \text{(12b)} \\
& \eta_i + \frac{1}{\underline{\alpha}_i^n} \sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}} \left[f(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}), \tilde{\mathbf{d}}) - \eta_i \right]^+ \leq \tau_i, && i \in \bar{\mathcal{L}}^n, && \text{(12c)} \\
& \sum_{j\in\mathcal{L}\setminus i} y_{ij}^s(\mathbf{d}) \leq r_i, && (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}^s,\ s \in \mathcal{S},\ i \in \mathcal{L}, && \text{(12d)} \\
& y_{ij}^s(\mathbf{d}) \geq 0, && (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}^s,\ s \in \mathcal{S},\ i, j \in \mathcal{L}, && \text{(12e)} \\
& (1\text{a}) - (1\text{d}), \alpha \in (0, 1], \boldsymbol{\eta} \geq \mathbf{0}. && && \text{(12f)}
\end{aligned}
$$

Next, we can show that given any fixed $(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{\eta}, \alpha)$, there exists a feasible solution for $y_{ij}^s(\mathbf{d})$ in problem (12) if and only if there is also a feasible mapping of the form $\bar{y}_{ij}^s(\mathbf{d}, \mathbf{z})$. The "if" direction is straighforward given that one can simply define $\bar{y}_{ij}^s(\mathbf{d}, \mathbf{z}) := y_{ij}^s(\mathbf{d})$. Regarding the "only if" part, one can use the feasible $\bar{y}_{ij}^s(\cdot)$ to define $y_{ij}^s(\mathbf{d}) := \bar{y}_{ij}^s(\mathbf{d}, |\mathbf{d} - \boldsymbol{\mu}^s|)$ and show that this is a feasible assignment in problem (12) based on:

$$
\begin{aligned}
\eta_i + \frac{1}{\alpha} \sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}} \left[f(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}), \tilde{\mathbf{d}}) - \eta_i \right]^+ &= \eta_i + \frac{1}{\alpha} \sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}} \left[f(\boldsymbol{x}, \boldsymbol{r}, \bar{\boldsymbol{y}}^{\tilde{s}}(\tilde{\mathbf{d}}, |\tilde{\mathbf{d}} - \boldsymbol{\mu}^s|), \tilde{\mathbf{d}}) - \eta_i \right]^+, \\
&\leq \eta_i + \frac{1}{\alpha} \sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}} \left[f(\boldsymbol{x}, \boldsymbol{r}, \bar{\boldsymbol{y}}^{\tilde{s}}(\tilde{\mathbf{d}}, \tilde{\mathbf{z}}), \tilde{\mathbf{d}}) - \eta_i \right]^+ \leq \tau_i,
\end{aligned}
$$

$$\sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s}\sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d}) = \sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s}\sum_{j\in\mathcal{L}\setminus i} \bar{y}^s_{ij}(\mathbf{d},|\mathbf{d}-\boldsymbol{\mu}^s|) \leq \sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s}\sum_{j\in\mathcal{L}\setminus i} \bar{y}^s_{ij}(\mathbf{d},\mathbf{z}) \leq r_i$$
$$\inf_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} y^s_{ij}(\mathbf{d}) = \inf_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \bar{y}^s_{ij}(\mathbf{d},|\mathbf{d}-\boldsymbol{\mu}^s|) \geq \inf_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \bar{y}^s_{ij}(\mathbf{d},\mathbf{z}) \geq 0\,.$$

We are left with employing classical moment problems duality techniques to reformulate $\sup_{\mathbb{Q}\in\mathcal{G}} \mathbb{E}_{\mathbb{Q}}\left[\left(f(\boldsymbol{x},\boldsymbol{r},\boldsymbol{y}^{\tilde{s}}(\tilde{\mathbf{d}}),\tilde{\mathbf{d}})-\eta_i\right)^+\right]$ as an infimum. Namely, one starts with the equivalent semi-infinite linear optimization problem:

$$\begin{aligned}
\max\ & \sum_{s\in\mathcal{S}} p_s \int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \left(\max\left\{0, -\eta_i, d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d},\mathbf{z})) - \eta_i\right\}\right) d\mathbb{Q}_s(\mathbf{d},\mathbf{z}) \\
\text{s.t.}\ & \int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} d\mathbb{Q}_s(\mathbf{d},\mathbf{z}) = 1, \quad s\in\mathcal{S}, \quad (\textit{dual multiplier } \pi^1_{si}\in\mathbb{R}) \\
& \int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \mathbf{d}\, d\mathbb{Q}_s(\mathbf{d},\mathbf{z}) = \boldsymbol{\mu}^s, \quad s\in\mathcal{S}, \quad (\textit{dual multiplier } \boldsymbol{\pi}^2_{si}\in\mathbb{R}^{|\mathcal{L}|}) \\
& \int_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s} \mathbf{u}\, d\mathbb{Q}_s(\mathbf{d},\mathbf{z}) \leq \boldsymbol{\nu}^s, \quad s\in\mathcal{S}. \quad (\textit{dual multiplier } \boldsymbol{\pi}^3_{si}\in\mathbb{R}^{|\mathcal{L}|}_+),
\end{aligned}$$

where $\mathbb{Q}_s$ denote the conditional probability distribution of $(\mathbf{d},\mathbf{z})$ given that $\tilde{s}=s$.

Letting $\pi^1_{si}\in\mathbb{R}$, $\boldsymbol{\pi}^2_{si}\in\mathbb{R}^{|\mathcal{L}|}$, and $\boldsymbol{\pi}^3_{si}\in\mathbb{R}^{|\mathcal{L}|}$ be dual variables associated with constraints of the above problem, we obtain the dual formulation as

$$\begin{aligned}
\min_{\pi^1_{si},\boldsymbol{\pi}^2_{si},\boldsymbol{\pi}^3_{si}\geq\mathbf{0}}\ & \sum_{s\in\mathcal{S}} \left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \\
\text{s.t.}\ & \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq 0, && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq -p_s\eta_i, && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}, \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq p_s\left(d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d},\mathbf{z})) - \eta_i\right), && (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s, s\in\mathcal{S}.
\end{aligned}$$

We thus obtain the formulation in the Lemma 2. □

B.5 Proof of Proposition 2.

By replacing $\bar{\mathcal{D}}^s$ with the bounded set $\bar{\mathcal{D}}^s_b$ in problem $\mathrm{P}_{\mathrm{ARO}}$, we obtain the following model:

$$\begin{aligned}
v^b_n := & \\
\inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha,\{\boldsymbol{y}^s(\cdot)\}_{s\in\mathcal{S}}\\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\geq\mathbf{0}}}\ & \alpha && \text{(13a)} \\
\text{s.t.}\ & \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, \quad i\in\mathcal{L}\setminus\bar{\mathcal{L}}^n, && \text{(13b)} \\
& \eta_i + \frac{1}{\underline{\alpha}^n_i}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, \quad i\in\bar{\mathcal{L}}^n, && \text{(13c)} \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq 0, \quad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s_b, s\in\mathcal{S}, i\in\mathcal{L}, && \text{(13d)} \\
& \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq -p_s\eta_i, \quad (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s_b, s\in\mathcal{S}, i\in\mathcal{L}, && \text{(13e)}
\end{aligned}$$

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq p_s\big(d_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^s_{ji}(\mathbf{d},\mathbf{z})) - \eta_i\big) \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s\in\mathcal{S}, i\in\mathcal{L}, \tag{13f}$$

$$\sum_{j\in\mathcal{L}\setminus i} y^s_{ij}(\mathbf{d},\mathbf{z}) \leq r_i, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s\in\mathcal{S}, i\in\mathcal{L}, \tag{13g}$$

$$y^s_{ij}(\mathbf{d},\mathbf{z}) \geq 0, \quad (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, s\in\mathcal{S}, i,j\in\mathcal{L}. \tag{13h}$$

$$(1a) - (1d), \alpha \in (0,1], \boldsymbol{\eta} \geq \mathbf{0}. \tag{13i}$$

The next lemma confirms that replacing $\bar{\mathcal{D}}^s$ with $\bar{\mathcal{D}}^s_b$ does not affect the optimal solution of problem P_{ARO}.

Lemma 6. *Problem P_{ARO} is equivalent to problem* (13)*, i.e., $v^*_n = v^b_n$ and the set of optimal solutions for $\boldsymbol{x}$ and $\boldsymbol{r}$ remains unchanged.*

Proof. First we note that $v^b_n \leq v^*_n$ because problem P_{ARO} has the same objective function but more constraints than problem (13).

Second, we prove $v^b_n \geq v^*_n$. Let $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\eta}}$, $\hat{\alpha}$, $\hat{\boldsymbol{\pi}}^1$, $\hat{\boldsymbol{\pi}}^2$, $\hat{\boldsymbol{\pi}}^3$, and $\hat{\boldsymbol{y}}^s(\mathbf{d},\mathbf{z})$, for all $s\in\mathcal{S}$, be a feasible solution to problem (13). It follows that the for all $s\in S, i\in\mathcal{L}, (\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s_b$, we have

$$\hat{\pi}^1_{si} + (\hat{\boldsymbol{\pi}}^2_{si})'\mathbf{d} + (\hat{\boldsymbol{\pi}}^3_{si})'\mathbf{z} \geq \max\left\{0, -p_s\hat{\eta}_i, p_s\left(d_i + \sum_{j\in\mathcal{L}\setminus i} \hat{y}^s_{ij}(\mathbf{d},\mathbf{z}) - \left(\hat{r}_i + \sum_{j\in\mathcal{L}\setminus i} \hat{y}^s_{ji}(\mathbf{d},\mathbf{z})\right) - \hat{\eta}_i\right)\right\}.$$

To simplify the notation, we rewrite the above formula as $f_{si}(\hat{\boldsymbol{\pi}},\mathbf{d},\mathbf{z}) \geq g_{si}(\hat{\boldsymbol{r}},\hat{\boldsymbol{\eta}},\hat{\boldsymbol{y}},\mathbf{d},\mathbf{z})$. Next, we can construct a feasible solution to problem P_{ARO} by letting $\check{\boldsymbol{x}} := \hat{\boldsymbol{x}}$, $\check{\boldsymbol{r}} := \hat{\boldsymbol{r}}$, $\check{\boldsymbol{\eta}} := \hat{\boldsymbol{\eta}}$, $\check{\alpha} := \hat{\alpha}$, $\check{\boldsymbol{\pi}}^1 := \hat{\boldsymbol{\pi}}^1$, $\check{\boldsymbol{\pi}}^2 := \hat{\boldsymbol{\pi}}^2$, $\check{\boldsymbol{\pi}}^3 := \hat{\boldsymbol{\pi}}^3$ and for all $s\in\mathcal{S}$,

$$\check{\boldsymbol{y}}^s(\mathbf{d},\mathbf{z}) := \begin{cases} \hat{\boldsymbol{y}}^s(\mathbf{d},\mathbf{z}) & \text{if } (\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b, \\ \hat{\boldsymbol{y}}^s(\mathbf{d},\min\{\mathbf{z},\bar{\mathbf{z}}^s\}) & \text{if } (\mathbf{d},\mathbf{z}) \notin \bar{\mathcal{D}}^s_b, \end{cases}$$

where $\bar{z}^s_i := \max\{\overline{d}^s_i - \mu^s_i, \mu^s_i - \underline{d}^s_i\}$ for all $i\in\mathcal{L}$, and where $\min\{\mathbf{z},\bar{\mathbf{z}}\}$ is applied termwise. We can verify that the constructed solution $(\check{\boldsymbol{x}},\check{\boldsymbol{r}},\check{\boldsymbol{\eta}},\check{\boldsymbol{\pi}},\check{\boldsymbol{y}})$ is feasible for problem P_{ARO} by first easily confirming that it is feasible with respect to all constraints besides (9d)-(9f). Regarding, (9d)-(9f), i.e. $f_{si}(\check{\boldsymbol{\pi}},\mathbf{d},\mathbf{z}) \geq g_{si}(\check{\boldsymbol{r}},\check{\boldsymbol{\eta}},\check{\boldsymbol{y}},\mathbf{d},\mathbf{z})$ for all i, s and $(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s$, we discussing the following two cases:

i) If $(\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}^s_b$, then $f_{si}(\check{\boldsymbol{\pi}},\mathbf{d},\mathbf{z}) = f_{si}(\hat{\boldsymbol{\pi}},\mathbf{d},\mathbf{z}) \geq g_{si}(\hat{\boldsymbol{r}},\hat{\boldsymbol{\eta}},\hat{\boldsymbol{y}},\mathbf{d},\mathbf{z}) = g_{si}(\check{\boldsymbol{r}},\check{\boldsymbol{\eta}},\check{\boldsymbol{y}},\mathbf{d},\mathbf{z})$, which follows from the definition of the constructed solution.

ii) If $(\mathbf{d},\mathbf{z}) \notin \bar{\mathcal{D}}^s_b$, then we have that $f(\check{\boldsymbol{\pi}},\mathbf{d},\mathbf{z}) = f(\hat{\boldsymbol{\pi}},\mathbf{d},\mathbf{z}) \geq f(\hat{\boldsymbol{\pi}},\mathbf{d},\min\{\mathbf{z},\bar{\mathbf{z}}\}) \geq g(\hat{\boldsymbol{r}},\hat{\boldsymbol{\eta}},\hat{\boldsymbol{y}},\mathbf{d},\min\{\mathbf{z},\bar{\mathbf{z}}\}) = g(\check{\boldsymbol{r}},\check{\boldsymbol{\eta}},\check{\boldsymbol{y}},\mathbf{d},\mathbf{z})$, where we used in order that $\check{\boldsymbol{\pi}} = \hat{\boldsymbol{\pi}}$, then that $\hat{\boldsymbol{\pi}}^3 \geq 0$, the fact that $(\hat{\boldsymbol{\pi}},\hat{\boldsymbol{r}},\hat{\boldsymbol{\eta}},\hat{\boldsymbol{y}})$ is feasible in (13) and that $(\mathbf{d},\min\{\mathbf{z},\bar{\mathbf{z}}\}) \in \bar{\mathcal{D}}^s_b$.

Because problem P_{ARO} and problem (13) have the same objective function, we conclude that $v^b_n \geq v^*_n$. Together with $v^b_n \leq v^*_n$, the proof is complete. □

In our proof of Proposition 2, for all $s\in\mathcal{S}$, we also exploit some convexity property of the following operator:

$$G^s(\boldsymbol{r},\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d},\mathbf{z}) := \begin{cases} 0 & \text{if } \mathcal{Y}^s(\alpha,\boldsymbol{\eta},\boldsymbol{\pi},\mathbf{d},\mathbf{z}) \neq \emptyset \\ \infty & \text{otherwise} \end{cases},$$

where set $\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z})$ is defined as:

$$\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z}) := \left\{ \boldsymbol{y} \in \mathbb{R}^{|\mathcal{L}|\times|\mathcal{L}|} : \begin{array}{lr} (13b) - (13c) & \\ \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq 0, & i \in \mathcal{L}, \\ \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq -p_s\eta_i, & i \in \mathcal{L}, \\ \pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq & \\ \qquad p_s(d_i + \sum_{j\in\mathcal{L}\setminus i} y_{ij} - (r_i + \sum_{j\in\mathcal{L}\setminus i} y_{ji}) - \eta_i) & i \in \mathcal{L}, \\ \sum_{j\in\mathcal{L}\setminus i} y_{ij} \leq r_i, & i \in \mathcal{L}, \\ y_{ij} \geq 0, & i, j \in \mathcal{L}, \end{array} \right\},$$

and which is considered empty if any constraint in $\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z})$ is violated.

Then, we can reorganize the constraints in problem (13) and equivalently rewrite it in the following general form:

$$\inf \left\{ \alpha \in (0,1] \;\middle|\; \begin{array}{c} \exists \boldsymbol{x}, \boldsymbol{r}, (1a) - (1d) \\ \inf_{\boldsymbol{\eta}\geq 0, \boldsymbol{\pi}} \max_s \sup_{(\mathbf{d},\mathbf{z})\in\bar{\mathcal{D}}^s_b} G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z}) \leq 0 \end{array} \right\} \tag{14}$$

Lemma 7. *$G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z})$ is convex jointly in $\mathbf{d}$, $\mathbf{z}$.*

Proof. Let $(\mathbf{d}_1, \mathbf{z}_1)$ and $(\mathbf{d}_2, \mathbf{z}_2)$ be arbitrary elements in $\mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|}$. For some fixed $\boldsymbol{r}$, α, $\boldsymbol{\eta}$, $\boldsymbol{\pi}$ and any $\lambda \in (0,1)$, we analyze the following two cases to evaluate $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_\lambda, \mathbf{z}_\lambda)$, where $(\mathbf{d}_\lambda, \mathbf{z}_\lambda) = \lambda(\mathbf{d}_1, \mathbf{z}_1) + (1-\lambda)(\mathbf{d}_2, \mathbf{z}_2)$.

First, when $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_1, \mathbf{z}_1) = \infty$ and/or $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_2, \mathbf{z}_2) = \infty$, we have $\lambda G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_1, \mathbf{z}_1) + (1-\lambda)G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_2, \mathbf{z}_2) \geq \infty \geq G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_\lambda, \mathbf{z}_\lambda)$, since the value of $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_\lambda, \mathbf{z}_\lambda)$ is either 0 or infinity.

Second, when $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_1, \mathbf{z}_1) = 0$ and $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_2, \mathbf{z}_2) = 0$, there exist $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ that satisfy all constraints defined in $\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_1, \mathbf{z}_2)$ and $\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_2, \mathbf{z}_2)$ respectively. Next, we can construct $\boldsymbol{y}_\lambda$ for $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_\lambda, \mathbf{z}_\lambda)$ by $\boldsymbol{y}_\lambda = \lambda \boldsymbol{y}_1 + (1-\lambda)\boldsymbol{y}_2$. We can easily verify that $\boldsymbol{y}_\lambda$ satisfies constraints in $\mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}, \mathbf{z})$. Thus, we have $\boldsymbol{y}_\lambda \in \mathcal{Y}^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_\lambda, \mathbf{z}_\lambda)$ such that $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}_\lambda, \mathbf{z}_\lambda) = 0$, which implies that

$$\lambda G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \mathbf{d}_1, \mathbf{z}_1) + (1-\lambda)G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \mathbf{d}_2, \mathbf{z}_2) = 0 \geq 0 = G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \mathbf{d}_\lambda, \mathbf{z}_\lambda).$$

Therefore, $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \mathbf{d}, \mathbf{z})$ is convex jointly in $\mathbf{d}$ and $\mathbf{z}$. □

We proceed with the proof of Proposition 2.

Recall that $\bar{\mathcal{D}}^s_b$ is a convex hull of $\{\mathbf{d}(\omega)\}_{\omega\in\Omega^s}$. By Lemma 7, we know that $G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \mathbf{d}, \mathbf{z})$ is a jointly convex function in $\mathbf{d}$ and $\mathbf{z}$. Hence, according to the theory of concave minimization (see, for instance, Benson, 1995), problem (14) can be replaced with

$$\inf \left\{ \alpha \in (0,1] \;\middle|\; \begin{array}{c} \exists \boldsymbol{x}, \boldsymbol{r}, (1a) - (1d) \\ \inf_{\boldsymbol{\eta}\geq 0, \boldsymbol{\pi}} \max_s \max_{\omega\in\Omega^s} G^s(\boldsymbol{r}, \alpha, \boldsymbol{\eta}, \boldsymbol{\pi}, \mathbf{d}(\omega), \mathbf{z}(\omega)) \leq 0 \end{array} \right\} \tag{15}$$

As a result, given a sample ω, the adaptive variables $\boldsymbol{y}(\mathbf{d}, \mathbf{z})$ reduce to $\boldsymbol{y}(\omega)$. This naturally gives rise to P_{VE} as an equivalent problem to P_{ARO} . □

B.6 MILP Reformulation of problem $\mathbf{P_{AARC}}$.

We present the reformulation of problem $\mathrm{P_{AARC}}$ by the following proposition.

Proposition 3. *Problem P_{AARC} can be solved by the following mixed-integer linear program:*

$$
\begin{aligned}
(P_{MILP}) \quad & v_n^{AARC} := \\
& \inf_{\substack{\boldsymbol{x},\boldsymbol{r},\boldsymbol{\eta},\alpha \\ \boldsymbol{\pi}^1,\boldsymbol{\pi}^2,\boldsymbol{\pi}^3\geq \mathbf{0},\boldsymbol{y}^0,\boldsymbol{y}^1,\boldsymbol{y}^2 \\ (\boldsymbol{w}^1,\boldsymbol{w}^2,\boldsymbol{w}^3,\boldsymbol{w}^4,\boldsymbol{w}^5)\geq \mathbf{0}}} \quad \alpha \\
s.t. \quad & \eta_i + \frac{1}{\alpha}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, && i\in\mathcal{L}\setminus\bar{\mathcal{L}}^n, \\
& \eta_i + \frac{1}{\underline{\alpha}_i^n}\sum_{s\in\mathcal{S}}\left(\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\boldsymbol{\mu}^s + (\boldsymbol{\pi}^3_{si})'\boldsymbol{\nu}^s\right) \leq \tau_i, && i\in\bar{\mathcal{L}}^n, \\
& \pi^1_{si} + (\boldsymbol{w}^{11}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{12}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{13}_{si} - \boldsymbol{w}^{14}_{si})'\boldsymbol{\mu}^s \geq 0, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \pi^1_{si} + (\boldsymbol{w}^{21}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{22}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{23}_{si} - \boldsymbol{w}^{24}_{si})'\boldsymbol{\mu}^s \geq -p_s\eta_i, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \pi^1_{si} + (\boldsymbol{w}^{31}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{32}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{33}_{si} - \boldsymbol{w}^{34}_{si})'\boldsymbol{\mu}^s \geq p_s(\sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ij} \\
& \quad - (r_i + \sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ji}) - \eta_i) && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \sum_{j\in\mathcal{L}\setminus i} y^{0s}_{ij} - \Big((\boldsymbol{w}^{41}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{42}_{si})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{43}_{si} - \boldsymbol{w}^{44}_{si})'\boldsymbol{\mu}^s\Big) \leq r_i, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& y^{0s}_{ij} + (\boldsymbol{w}^{51}_{sij})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{52}_{sij})'\overline{\mathbf{d}}^s - (\boldsymbol{w}^{53}_{sij} - \boldsymbol{w}^{54}_{sij})'\boldsymbol{\mu}^s \geq 0, && s\in\mathcal{S}, i,j\in\mathcal{L}, \\
& \boldsymbol{w}^{11}_{si} - \boldsymbol{w}^{12}_{si} - (\boldsymbol{w}^{13}_{si} - \boldsymbol{w}^{14}_{si}) \leq \boldsymbol{\pi}^2_{si}, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \boldsymbol{w}^{21}_{si} - \boldsymbol{w}^{22}_{si} - (\boldsymbol{w}^{23}_{si} - \boldsymbol{w}^{24}_{si}) \leq \boldsymbol{\pi}^2_{si}, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& w^{31}_{sil} - w^{32}_{sil} - (w^{33}_{sil} - w^{34}_{sil}) \leq v^2_{sil} - \sum_{j\in\mathcal{L}\setminus i} p_s(y^{1s}_{ijl} - y^{1s}_{jil}), && s\in\mathcal{S}, l,i\in\mathcal{L}, l\neq i, \\
& w^{31}_{sil} - w^{32}_{sil} - (w^{33}_{sil} - w^{34}_{sil}) \leq v^2_{sil} - \sum_{j\in\mathcal{L}\setminus i} p_s(y^{1s}_{ijl} - y^{1s}_{jil}) - p_s, && s\in\mathcal{S}, l,i\in\mathcal{L}, l = i, \\
& \boldsymbol{w}^{41}_{si} - \boldsymbol{w}^{42}_{si} - (\boldsymbol{w}^{43}_{si} - \boldsymbol{w}^{44}_{si}) \leq -\sum_{j\in\mathcal{L}\setminus i} \boldsymbol{y}^{1s}_{ij}, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \boldsymbol{w}^{51}_{sij} - \boldsymbol{w}^{52}_{sij} - (\boldsymbol{w}^{53}_{sij} - \boldsymbol{w}^{54}_{sij}) \leq \boldsymbol{y}^{1s}_{ij}, && s\in\mathcal{S}, i,j\in\mathcal{L}, \\
& \boldsymbol{w}^{13}_{si} + \boldsymbol{w}^{14}_{si} \leq \boldsymbol{\pi}^3_{si}, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \boldsymbol{w}^{23}_{si} + \boldsymbol{w}^{24}_{si} \leq \boldsymbol{\pi}^3_{si}, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \boldsymbol{w}^{33}_{si} + \boldsymbol{w}^{34}_{si} \leq \boldsymbol{\pi}^3_{si} - \sum_{j\in\mathcal{L}\setminus i} p_s(\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji}), && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \boldsymbol{w}^{43}_{si} + \boldsymbol{w}^{44}_{si} \leq -\sum_{j\in\mathcal{L}\setminus i} \boldsymbol{y}^{2s}_{ij}, && s\in\mathcal{S}, i\in\mathcal{L}, \\
& \boldsymbol{w}^{53}_{sij} + \boldsymbol{w}^{54}_{sij} \leq \boldsymbol{y}^{2s}_{ij}, && s\in\mathcal{S}, i,j\in\mathcal{L}, \\
& (1\text{a}) - (1\text{d}), \alpha\in(0,1], \boldsymbol{\eta}\geq \mathbf{0}.
\end{aligned}
$$

Proof. For all $s \in \mathcal{S}$, we represent the adaptive decisions by an affine functions $\boldsymbol{y}^s(\cdot) \in \mathscr{A}$. For example, we can rewrite constraint (11f) as

$$\pi^1_{si} + (\boldsymbol{\pi}^2_{si})'\mathbf{d} + (\boldsymbol{\pi}^3_{si})'\mathbf{z} \geq$$
$$p_s \left(d_i + \sum_{j \in \mathcal{L} \setminus i} \left(y^{0s}_{ij} + (\boldsymbol{y}^{1s}_{ij})'\mathbf{d} + (\boldsymbol{y}^{2s}_{ij})'\mathbf{z} \right) - \left(r_i + \sum_{j \in \mathcal{L} \setminus i} \left(y^{0s}_{ji} + (\boldsymbol{y}^{1s}_{ji})'\mathbf{d} + (\boldsymbol{y}^{2s}_{ji})'\mathbf{z} \right) \right) - \eta_i \right),$$
$$(\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}_s, s \in \mathcal{S}, i \in \mathcal{L}.$$

We reorganize the terms in the above constraint:

$$\pi^1_{si} + \min_{(\mathbf{d},\mathbf{z}) \in \bar{\mathcal{D}}_s} \left\{ \left(\boldsymbol{\pi}^2_{si} - \sum_{j \in \mathcal{L} \setminus i} p_s (\boldsymbol{y}^{1s}_{ij} - \boldsymbol{y}^{1s}_{ji}) \right)' \mathbf{d} + \left(\boldsymbol{\pi}^3_{si} - \sum_{j \in \mathcal{L} \setminus i} p_s (\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji}) \right)' \mathbf{z} - p_s d_i \right\} \geq$$
$$p_s \left(\sum_{j \in \mathcal{L} \setminus i} y^{0s}_{ij} - (r_i + \sum_{j \in \mathcal{L} \setminus i} y^{0s}_{ji}) - \eta_i \right), \qquad s \in \mathcal{S}, i \in \mathcal{L}.$$

We now focus on the above minimization subproblem. For all $s \in \mathcal{S}, i \in \mathcal{L}$, the subproblem can be rewritten as follows:

$$\begin{aligned}
\min_{\mathbf{d},\mathbf{z}} \; & \left(\boldsymbol{\pi}^2_{si} - \sum_{j \in \mathcal{L} \setminus i} p_s (\boldsymbol{y}^{1s}_{ij} - \boldsymbol{y}^{1s}_{ji}) \right)' \mathbf{d} + \left(\boldsymbol{\pi}^3_{si} - \sum_{j \in \mathcal{L} \setminus i} p_s (\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji}) \right)' \mathbf{z} - p_s d_i \\
\text{s.t.} \; & \underline{\mathbf{d}}^s \leq \mathbf{d} \leq \overline{\mathbf{d}}^s, \qquad \text{(dual multipliers } \boldsymbol{w}^{31}_{si}, \boldsymbol{w}^{32}_{si} \in \mathbb{R}^{|\mathcal{L}|}_+) \\
& -\mathbf{z} \leq \mathbf{d} - \boldsymbol{\mu}^s \leq \mathbf{z}, \qquad \text{(dual multipliers } \boldsymbol{w}^{33}_{si}, \boldsymbol{w}^{34}_{si} \in \mathbb{R}^{|\mathcal{L}|}_+)
\end{aligned}$$

By the strong duality, we present the dual formulation of the above subproblem as:

$$\begin{aligned}
\max_{(\boldsymbol{w}^{31}_{si}, \boldsymbol{w}^{32}_{si}, \boldsymbol{w}^{33}_{si}, \boldsymbol{w}^{34}_{si}) \geq \mathbf{0}} \; & (\boldsymbol{w}^{31}_{si})'\underline{\mathbf{d}}^s - (\boldsymbol{w}^{32}_{si})'\overline{\mathbf{d}}^s - \left(\boldsymbol{w}^{33}_{si} - \boldsymbol{w}^{34}_{si} \right)' \boldsymbol{\mu}^s \\
\text{s.t.} \; & w^{31}_{sil} - w^{32}_{sil} - (w^{34}_{sil} - w^{33}_{sil}) \leq v^2_{sil} - \sum_{j \in \mathcal{L} \setminus i} p_s (y^{1s}_{ijl} - y^{1s}_{jil}), \qquad l \in \mathcal{L}/i, \\
& w^{31}_{sii} - w^{32}_{sii} - (w^{34}_{sii} - w^{33}_{sii}) \leq v^2_{sii} - \sum_{j \in \mathcal{L} \setminus i} p_s (y^{1s}_{iji} - y^{1s}_{jii}) - p_s, \\
& \boldsymbol{w}^{33}_{si} + \boldsymbol{w}^{34}_{si} \leq \boldsymbol{\pi}^3_{si} - \sum_{j \in \mathcal{L} \setminus i} p_s (\boldsymbol{y}^{2s}_{ij} - \boldsymbol{y}^{2s}_{ji}).
\end{aligned}$$

For other constraint sets in problem $\mathrm{P}_{\mathrm{AARC}}$, we can perform the similar conversion. Therefore, combining all parts together, we can obtain the MILP reformulation. □

C PRNDP-NE Formulation

According to previous studies (see, for instance, Rawls and Turnquist, 2010), we formulate the predisaster relief network design problem without equity (PRNDP-NE) as follows:

$$\text{(PRNDP-NE)} \quad \min_{\boldsymbol{x}, \boldsymbol{r}, \tilde{\boldsymbol{u}}, \tilde{\boldsymbol{y}}} \sup_{\mathbb{P} \in \mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[\sum_{i \in \mathcal{L}} \tilde{u}_i \right] \tag{16}$$

$$\text{s.t.} \quad (1a) - (1d), (2b) - (2d).$$

Similar to Lemma 2, the PRNDP-NE with $\mathcal{F}$, i.e., $\mathcal{F} = \prod_{\tilde{\mathbf{d}}, \tilde{s}} \mathcal{G}$, is equivalent to the following optimization problem:

$$\min_{\substack{\boldsymbol{x}, \boldsymbol{r}, \{\boldsymbol{y}^s(\cdot)\}_{s \in \mathcal{S}} \\ \boldsymbol{\pi}^1, \boldsymbol{\pi}^2, \boldsymbol{\pi}^3 \geq \mathbf{0}}} \sum_{s \in \mathcal{S}} \left(v_s^1 + (\boldsymbol{\pi}_s^2)' \boldsymbol{\mu}^s + (\boldsymbol{\pi}_s^3)' \boldsymbol{\nu}^s \right) \tag{17a}$$

$$\text{s.t.} \quad \pi_s^1 + (\boldsymbol{\pi}_s^2)'\mathbf{d} + (\boldsymbol{\pi}_s^3)'\mathbf{z} \geq p_s \sum_{i \in \mathcal{L}} f(\boldsymbol{x}, \boldsymbol{r}, \boldsymbol{y}^s(\mathbf{d}, \mathbf{z}), \mathbf{d}) \qquad (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}_s, s \in \mathcal{S}, \tag{17b}$$

$$\sum_{j \in \mathcal{L} \setminus i} y_{ij}^s(\mathbf{d}, \mathbf{z}) \leq r_i, \qquad (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}_s, s \in \mathcal{S}, i \in \mathcal{L}, \tag{17c}$$

$$y_{ij}^s(\mathbf{d}, \mathbf{z}) \geq 0, \qquad (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}_s, s \in \mathcal{S}, i, j \in \mathcal{L}, \tag{17d}$$

$$(1a) - (1d),$$

where each $y_{ij}^s : \mathbb{R}^{|\mathcal{L}|} \times \mathbb{R}^{|\mathcal{L}|} \to \mathbb{R}$, and where π_s^1, $\boldsymbol{\pi}_s^2$ and $\boldsymbol{\pi}_s^3$ are the dual variables associated with first, second, and third constraints that define $\mathcal{G}$.

Since constraint (17b) is a sum of max function, by Definition 1, it can be written as

$$\pi_s^1 + (\boldsymbol{\pi}_s^2)'\mathbf{d} + (\boldsymbol{\pi}_s^3)'\mathbf{z} \geq p_s \sum_{i \in \mathcal{A}} \left(d_i + \sum_{j \in \mathcal{L} \setminus i} y_{ij}^s(\mathbf{d}, \mathbf{z}) - (r_i + \sum_{j \in \mathcal{L} \setminus i} y_{ji}^s(\mathbf{d}, \mathbf{z})) \right), A \in P(\mathcal{L}), (\mathbf{d}, \mathbf{z}) \in \bar{\mathcal{D}}_s, s \in \mathcal{S},$$

where $P(\mathcal{L})$ is the power set of $\mathcal{L}$. Note that we can apply the affine decision rule and obtain an affinely adjustable robust counterpart of problem (17). Finally, problem (17) can also be transformed into a MILP via standard techniques in robust optimization.

D Data, Concepts and Results of the Numerical Study

D.1 Input Parameters for the Numerical Study.

We conduct a case study using the earthquake that happened at Yushu County in Qinghai Province, PR China in 2010. This case represents an affected area by a network consisting of 13 locations and 15 links. For deterministic parameters (e.g., cost, capacity, donations, etc.), we refer to Ni et al. (2018) and show them in Table 6. As stated by Kanamori (1978): "Since the physical process underlying an earthquake is very complex, we cannot express every detail of an earthquake by a single parameter. However, it would be convenient if we could find a single number that represents the overall physical size of an earthquake." Thus, we construct the scenario-wise ambiguity set based on the *Richter scale* ($\mathrm{M_L}$) that is a commonly used measure of the strength of earthquakes (Richter, 1935). Specifically, the *Richter scale* is divided into nine scales: felt slightly by some people less than $\mathrm{M_L}2.5$, often felt by people above $\mathrm{M_L}2.5$, can cause damage if above $\mathrm{M_L}5.0$. Therefore, we generate 5 scenarios with equal probabilities (i.e., $S = \{1, 2, 3, 4, 5\}$), in which each scenario corresponds to a scale above $\mathrm{M_L}5.0$. Intuitively, the greater the scale of the earthquake, the more demand for relief supplies there will be, and more areas will be affected, so we take this into account when generating demands-related parameters. Specifically, the underlying model is a joint distribution over $(\tilde{s}, \tilde{\mathbf{d}})$ with $\tilde{s} \sim \{1, \ldots, 5\}$ uniformly, and each $\tilde{d}_i \sim N(\bar{\mu}_i^{\tilde{s}}, \bar{\sigma}_i^{\tilde{s}}, 0, +\infty)$ for all $i \in \mathcal{L}$, where N is a truncated normal distribution, and where $\bar{\mu}_i^s = 25 + 25s$ and $\bar{\sigma}_i^s = 0.1\bar{\mu}_i^s$, We also let $\tau_i = 0.02 \times \sum_{s \in S} \bar{\mu}_i^s$ for all $i \in \mathcal{L}$.

Similar to Ni et al. (2018), we first generate 50 samples from the joint distribution. We then let $\hat{p}_s$, $\hat{\boldsymbol{\mu}}^s$, $\hat{\boldsymbol{\nu}}^s$, $\hat{\boldsymbol{\sigma}}^s$ $\hat{\bar{\mathbf{d}}}^s$, $\hat{\underline{\mathbf{d}}}^s$ be, for each s, the empirical frequency of the scenario in the observed samples, the empirical conditional mean, conditional mean absolute deviation, conditional standard deviation, conditional minimum, and conditional maximum in the 50 observations. We also let $\hat{\boldsymbol{\mu}}$, $\hat{\boldsymbol{\nu}}$, $\hat{\bar{\mathbf{d}}}$, $\hat{\underline{\mathbf{d}}}$ be the empirical (unconditional) mean, mean absolute deviation, minimum, and maximum in the 50 observations. With these statistics in hand, we can construct the RSO model, the DRO model, and the SAA model as follows.

- For the RSO model, the ambiguity set $\mathcal{F}$ is defined using $(\hat{\boldsymbol{\mu}}^s, \hat{\boldsymbol{\nu}}^s, \hat{\bar{\mathbf{d}}}^s, \hat{\underline{\mathbf{d}}}^s)$.
- For the DRO model, the ambiguity set $\mathcal{F}$ can be regarded as a special case containing only one scenario, which is defined using $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\nu}}, \hat{\underline{\mathbf{d}}}, \hat{\bar{\mathbf{d}}})$.
- For the SAA model, we consider 100 samples drawn from $\tilde{s} \sim$ Mutlinoulli$(\hat{p}_1, \ldots, \hat{p}_5)$ and each $d_i \sim N(\hat{\mu}_i^{\tilde{s}}, \hat{\sigma}_i^{\tilde{s}}, 0, +\infty)$ for all $i \in L$, where Mutlinoulli is a multinoulli distribution.

After solving all the above models, we can obtain predisaster deployment decisions $(\boldsymbol{x}^*, \boldsymbol{r}^*)$ for each of these models. To evaluate the performance of such solutions, we generate 1,000 out-of-sample demands drawn from $\tilde{s} \sim$ Mutlinoulli$(1, \ldots, 5)$ and each $d_i \sim N(\bar{\mu}_i^{\tilde{s}}, \bar{\sigma}_i^{\tilde{s}}, 0, +\infty)$ for all $i \in L$. Moreover, to capture the possibility that the out-of-sample distribution may deviate from the underlying distribution, for each $s \in \mathcal{S}$, we add a perturbation Δ to the mean of the demand, i.e., $\bar{\boldsymbol{\mu}}^s_{out} = (1+\Delta)\bar{\boldsymbol{\mu}}^s$ and $\bar{\boldsymbol{\delta}}^s_{out} = 0.1\bar{\boldsymbol{\mu}}^s_{out}$, where Δ is enumerated from -0.1 to 0.1 with a step size of 0.05. To evaluate the impact of different donation levels, we consider R could be any value in the set {1,700, 1,750, . . . , 1,900}.

Table 6: Input parameters

Number of locations	$\lvert\mathcal{L}\rvert = 13$												
Location index i	1	2	3	4	5	6	7	8	9	10	11	12	13
Fixed cost c_i	203	193	130	117	292	174	130	157	134	161	234	220	170
Facility capacity	$M = 800$												
Total supplies	$R = 1,800$												
Total Budget	$B = 1,000$												

D.2 Concepts of equity indices.

Here we list the definitions and computational methods of MSG, RMD, VAR, SPAD and Gini in detail, as follows:

- *Maximum Shortage Gap*(MSG): This is the difference between the most and least deprived parties in terms of shortage ($\max_{i\in\mathcal{L}} u_i - \min_{i\in\mathcal{L}} u_i$), which focuses on two extreme cases.
- *Relative Mean Deviation*(RMD): This is the absolute deviation from the average shortage ($\sum_{i\in\mathcal{L}} |u_i - \bar{u}|$, where $\bar{u} = \frac{\sum_{i\in\mathcal{L}} u_i}{|\mathcal{L}|}$). Compared with MSG which only considers two extremes, RMD considers other levels of shortages as well.
- *Variance*(VAR): This is the average squared deviation from the average shortage ($\frac{\sum_{i\in\mathcal{L}}(u_i-\bar{u})^2}{|\mathcal{L}|}$).
- *Sum of Pairwise Absolute Differences*(SPAD): This is the sum of absolute differences between the shortages of all pairs of demand locations ($\sum_{i,j\in\mathcal{L}} |u_i - u_j|$).
- *Gini coefficient*(Gini): This index measures the inequity among values of a frequency distribution ($\frac{\sum_{i,j\in\mathcal{L}}|u_i-u_j|}{2|\mathcal{L}|\sum_{i\in\mathcal{L}} u_i}$). The coefficient ranges from 0 to 1, with 0 representing perfect equity and 1 representing perfect inequity.

D.3 Performance of AARC

We demonstrate the performance of AARC on small instances of the Earthquake case study. For simplicity, we only consider one scenario in the scenario-wise ambiguity set. To generate parameters related to demand, we randomly sample according to the sampling range in Table 7. For other parameters, we consider the same setup as the Earthquake case study.

Table 8 reports statistics on the suboptimality gaps observed in 100 random instances with sizes $|\mathcal{L}| = \{5, 6, 7\}$. One remarks that the suboptimality gaps are quite small, namely with a 95%-VaR of less than 1% and a maximum gap of 8%. Furthermore, suboptimality gap does not seem to increase with the size of the instance.

In addition, we also compare in Table 9 the solution time of AARC, VE, and SAA (with 1,000 samples). We choose the termination criteria to be whether the optimality gap is less than 0.01%, or the CPU time exceeds 7,200 seconds. Obviously, the performance of the AARC is significantly faster to solve than VE and SAA. In particular, VE starts taking more than 2 hours to solve for $|\mathcal{L}| \geq 9$, while AARC still returns a solution within 5 seconds. One also notices that the solution time of SAA increases more rapidly than AARC when the number of locations increases.

Table 7: Parameter setup related to demand at each location $i \in \mathcal{L}$

Parameter	Sampling range
Lower bound of demand $\underline{d}_i$	(0,50)
Upper bound of demand $\overline{d}_i$	$\underline{d}_i$+(0,100)
Mean of demand μ_i	($\underline{d}_i$,$\overline{d}_i$)
Mean absolute deviation of demand ν_i	(0,μ_i)
Threshold on demand τ_i	$0.1 \times \mu_i$

Table 8: Statistics of AARC suboptimality

$\|\mathcal{L}\|$	Minimum	Median	VaR@95%	VaR@99%	Maximum
5	0.00%	0.00%	0.88%	4.14%	6.34%
6	0.00%	0.00%	0.51%	2.78%	7.98%
7	0.00%	0.00%	0.64%	3.75%	7.25%

Table 9: Average CPU Time (in sec)

$\|\mathcal{L}\|$	AARC	VE	SAA
5	1.01	8.52	92.19
6	1.40	47.32	184.06
7	1.57	519.38	299.37
8	1.95	5,359.57	501.43
9	2.71	$> 7,200$	635.81
10	2.66	$> 7,200$	807.51
11	2.89	$> 7,200$	1,005.20
12	3.66	$> 7,200$	1,808.13
13	4.39	$> 7,200$	3,165.41

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