Les Cahiers du GERAD ISSN: 0711–2440

A successive linear programming algorithm with non-linear time series for the reservoir management problem

C. Gauvin, E. Delage,
M. Gendreau

G–2017–13

March 2017

GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

A successive linear programming algorithm with non-linear time series for the reservoir management problem

Charles Gauvin [a,b,d]
Erick Delage [a,c]
Michel Gendreau [b,d]

[a] GERAD

[b] Department of Mathematics and Industrial Engineering, Polytechnique Montréal, Montréal (Québec) Canada H3C 3A7

[c] HEC Montréal, Montréal (Québec) Canada, H3T 2A7

[d] CIRRELT

charles.gauvin@polymtl.ca
erick.delage@hec.ca
michel.gendreau@cirrelt.net

March 2017

Les Cahiers du GERAD

G–2017–13

Abstract: This paper proposes a multi-stage stochastic programming formulation based on affine decision rules for the reservoir management problem. Our approach seeks to find a release schedule that balances flood control and power generation objectives while considering realistic operating conditions as well as variable water head. To deal with the non-convexity introduced by the variable water head, we implement a simple, yet effective, successive linear programming algorithm. We also introduce a novel non-linear inflow representation that captures serial correlation of arbitrary order. We test our method on a real river system and discuss policy implications. Our results namely show that our method can decrease flood risk compared to historical decisions, albeit at the cost of reduced final storages.

Keywords: Mathematical programming, stochastic processes, forecasting, risk analysis

Acknowledgments: The authors would like to thank Grégory Émiel, Louis Delorme, Pierre-Marc Rondeau, Sara Séguin and Pierre-Luc Carpentier. This research was supported by the Natural Sciences and Engineering Reasearch Council of Canada (NSERC) and Hydro-Québec through the Industrial Research Chair on the Stochastic Optimization of Electricity Generation and NSERC grant 386416-2010.

1 Nomenclature

1.1 Sets and parameters

T	Total number of periods
L	Length of look-ahead period ($L \leq T$)
$\mathbb{T}_{t,L}$	Look-ahead period of L periods starting at time t
$\mathbb{L}$	Lead times
$\mathbb{K}^{prod.\,ref.}$	Piecewise linear segments approximating the reference production curve
$\mathbb{K}^{flood\,dmg.}$	Piecewise linear segments approximating the flood damage curve
$\mathbb{K}^{f.val.}$	Piecewise linear segments approximating the storage value function curve
I	Set of plants
I^{evac}	Set of plants with evacuation curve constraints
J	Set of reservoirs
$j^+(i)$, $j^-(i)$	Unique reservoir upstream or downstream of plant i, respectively
$\mathcal{H}_i^{ref}$	Reference water head at plant i
$\mathcal{N}_{jt}^{ref}$	Reference water level at reservoir j
λ_{il}	Fraction of the releases from plant i reaching the unique downstream reservoir after l periods
δ_i^{min}, δ_i^{max}	Minimum and maximum water delays for plant i
γ_j	Relative importance of flood damages at reservoir j
ϕ	Relative importance of power generation objectives compared to flood control
α_{jt}	Average proportion of the total inflows entering reservoir j at time t
$\underline{l}_i$, $\bar{l}_i$	Upper and lower bounds on total spillage at plant i
$\underline{r}_i$, $\overline{r}_i$	Upper and lower bounds on releases at plant i
$\underline{f}_i$, $\overline{f}_i$	Upper and lower bounds on total releases/total flow at plant i
$\overline{\Delta}_i$	Maximum flow variation between periods at plant i
$\underline{s}_j$, $\overline{s}_j$	Upper and lower bounds on storage at reservoir j
l_i^{Δ}, l_i^0	Parameters for the linear approximation of the evacuation curve at plant i
p_{ik}^{Δ}, p_{ik}^0	Parameters for the k^{th} segment of piecewise linear approximation of the reference production function for plant i
n_j^{Δ}, n_j^0	Parameters for the linear approximation of the water level curve for reservoir j
e_{jk}^{Δ}, e_{jk}^0	Parameters for the k^{th} segment of piecewise linear approximation of the flood penalty at reservoir j
s_{jk}^{Δ}, s_{jk}^0	Parameters for the k^{th} segment of piecewise linear approximation of the final water value function at reservoir j
ν_t	Price of 1 MWh during time t
η	Number of hours per period

1.2 Decision variables

$\mathcal{R}_{it}$	Releases at plant i during time t
$\mathcal{L}_{it}$	Spills at plant i during time t
$\mathcal{F}_{it}$	Total flow at plant i during time t
$\mathcal{S}_{jt}$, $\bar{\mathcal{S}}_{jt}$	Storage and value of storage at reservoir j at the end of time t
$\mathcal{E}_{jt}$, $\bar{\mathcal{E}}_{jt}$	Floods and flood damages at reservoir j at the end of time t
$\mathcal{H}_{it}$	Relative water head at plant i and time t
$\mathcal{P}_{it}$, $\mathcal{P}_{it}^{total}$	Reference and real production at plant i and time t
$\mathcal{N}_{jt}$	Water level at reservoir j and time t
$\mathcal{O}_t^{floods}$	Cost associated with floods at time t
$\mathcal{O}_t^{prod}$	Cost associated with electricity generation and final value of storage at time t

1.3 Random variables

ξ_t	Total inflows over all reservoirs during time t
ξ_{jt}	Inflows at reservoir j during time t
ζ_t	Logged (total) inflows during time t
ϱ_t	Residuals at time t
$\mathcal{G}_t$	Information known at time t
Ξ_t	Dynamic uncertainty set given $\mathcal{G}_t$
$\hat{\Xi}_t$	Exterior polyhedral approximation of Ξ_t

2 Introduction

This paper considers the optimal operation of a multi-reservoir hydro-electrical system over a given time horizon subject to uncertainty on inflows. We seek to balance hydro-electrical production and flood control while respecting a host of operating constraints on storages, spills, releases and water balance. Our model explicitly considers water delays as well as variable water head. In addition, we consider inflow persistence through ARIMA time series of *arbitrary* order.

As mentioned in [1], this stochastic multi-stage non-convex problem is extremely challenging to solve. The variable water head and the associated non-convexity make it impractical to find a globally optimal solution while the multi-dimensional nature of the problem often requires various simplifications that may lead to policies of limited practical use [2].

To solve this problem, we propose a stochastic program based on affine decision rules, non-linear ARIMA time series and a successive linear programming (SLP) algorithm. We incorporate our lightweight model in a rolling horizon framework and test it with historical inflows. Our approach builds on previous work [3, 4], but makes important improvements in terms of modeling and empirical results, namely regarding flood reduction.

Following the work of [5, 6], affine decision rules have gained some attention for reservoir management problems and other energy-related problems [3, 7]. Although they do not guarantee optimality in general, these simple parametric policies often lead to good-quality solutions that can be obtained very efficiently.

Unlike competing methods such as traditional stochastic dynamic programming (SDP) [8] and various other extensions such as sampling stochastic dynamic programming [9] and SDP based on improved discretization schemes [10], affine decision rules do not require discretizing the state-space, which results in important computational gains.

As illustrated in [4], affine decision rules also make it possible to consider long water delays as well as highly persistent inflows without increasing the computational burden as would be the case of algorithms belonging to the family of SDP. Considering these long inflow delays can be essential to avoid overly optimistic solutions in the face of floods [4, 8, 11].

Contrary to scenario-tree based stochastic programming and stochastic dual dynamic programming (SDDP), these decision rules also provide easily implementable policies that can be computed off-line and used by operators for simulations.

This paper also extends [4] and [7] by introducing a new inflow representation based on a simple non-linear time series model. Although this leads to a non-convex support, the resulting model has nice statistical interpretation and good forecasting ability.

By exploiting this representation along with affine decision rules, we are able to obtain a compact bilinear expression for the objective function. In order to tackle the non-convexity, we implement a simple SLP algorithm. These algorithms have been successfully utilized in hydro-electrical reservoir management problems in the context of stochastic programs based on scenario-trees [12], distributionally robust optimization [13] as well as at the operational level for deterministic and stochastic optimization, namely at Hydro-Québec. These experimental results suggest that this simple algorithm can provide good quality solutions since the objective is bilinear and therefore well approximated by a linear first-order Taylor approximation in a restricted neighborhood.

Considering variable water head is important since it captures the fact that a better production schedule can produce more electricity by using less water than the corresponding optimal plan with fixed head [14]. This is also an important consideration from an operational perspective in various regions such as Qubec where cascaded river systems are widespread. Evaluating water head is an important advantage of our method compared with SDDP since this method relies on convexity assumptions to obtain hyperplanes bounding the optimal cost-to-go functions [15, 16, 17].

The paper is structured as follows. We present the deterministic reservoir management problem in Section 3. We then discuss our non-linear inflow representation and the associated non-convex uncertainty set in Section 4. Section 5 presents the multi-stage stochastic program based on affine decision rules as well as the SLP algorithm used to solve it approximately. We then present the river system used for our numerical experiments in Section 6. Section 7 discusses results and Section 8 draws concluding comments.

3 Deterministic formulation

3.1 Daily river operations

River operators must determine the average water release that will flow through the turbines of the powerhouse i during day $\tau = t + l$ where $\tau \in \mathbb{T}_{t,L} = \{t, \cdots, t + L - 1\}$.[1] Because of plant specificities and other physical limitations, the total releases are required to reside within some closed interval:

$$\underline{r}_i \leq \mathcal{R}_{i\tau} \leq \bar{r}_i \qquad \forall i \in I, \tau \in \mathbb{T}_{t,L}. \tag{1}$$

Based on physical characteristics of the sites, a certain amount of water can also be unproductively spilled:

$$\underline{l}_i \leq \mathcal{L}_{i\tau} \leq \bar{l}_i \qquad \forall i \in I, \tau \in \mathbb{T}_{t,L}. \tag{2}$$

Certain sites $i \in I^{evac} \subset I$ can also evacuate water through a separate spillway. Due to the physical structure of these evacuators, the maximal amount of water that can be physically spilled is bounded by a function of $\mathcal{S}_{j\tau}$, the storage (hm^3) in the unique upstream reservoir $j^-(i)$ at the end of time τ. We approximate these functions with affine functions parametrized by $l_i^\Delta, l_i^0 \in \mathbb{R}$ to obtain:

$$\mathcal{L}_{i\tau} \leq l_i^\Delta \mathcal{S}_{j^-(i)\tau} + l_i^0, \qquad \forall i \in I^{evac}, \tau \in \mathbb{T}_{t,L}. \tag{3}$$

We define $\mathcal{F}_{i\tau} = \mathcal{R}_{i\tau} + \mathcal{L}_{i\tau}$ as the total flow (hm^3/day) $\forall i \in I, \tau \in \mathbb{T}_{t,L}$, which is simply the sum of unproductive spillage and releases. These quantities are bounded for navigation and recreation purposes, environmental needs as well as to ensure smooth ice formation during the winter months:

$$\underline{f}_i \leq \mathcal{F}_{i\tau} \leq \bar{f}_i \qquad \forall i \in I, \tau \in \mathbb{T}_{t,L}. \tag{4}$$

To avoid plant damages, we also ensure that the total flow does not change excessively from one day to the next:

$$|\mathcal{F}_{i\tau} - \mathcal{F}_{i\tau-1}| \leq \bar{\Delta}_i \qquad \forall i \in I, \tau \in \mathbb{T}_{t,L}.^{2} \tag{5}$$

We require that the final water storage at the end of time τ at reservoir j lie within fixed bounds:

$$\underline{s}_j \leq \mathcal{S}_{j\tau} \leq \bar{s}_j \qquad \forall j \in J, \tau \in \mathbb{T}_{t,L}. \tag{6}$$

Given very wet conditions, it is possible that the upper bound constraint (6) is violated. We therefore introduce the non-negative variable $\mathcal{E}_{j\tau} \geq 0$ representing floods and modify the second inequality to: $\mathcal{S}_{j\tau} - \mathcal{E}_{j\tau} \leq \bar{s}_j, \forall j \in J, \tau \in \mathbb{T}_{t,L}$. [3]

[1] For the level of granularity considered by our model (daily decisions), the different turbines are aggregated and it is assumed that no maintenance occurs so that all turbines are available for power generation.

[2] $\mathcal{F}_{i,t+l}$ are already known at time t for any $l \leq 0$.

[3] For sake of simplicity, it is assumed that floods remain in the reservoir. Although physically possible in some cases when either $\underline{s}_j > 0$ or reservoirs represent"useful" storages, we do not consider lower bounds violations.

We model the flood severity ($) as a piecewise linear increasing convex function of floods, which reflects the assumption that greater floods have increasing marginal consequences. We specifically introduce the decision variable $\bar{\mathcal{E}}_{j\tau} \geq 0$ and impose:

$$\bar{\mathcal{E}}_{j\tau} \geq e^{\Delta}_{jk}\mathcal{E}_{j\tau} + e^{0}_{jk}, \forall k \in \mathbb{K}^{flood\, dmg.}, \tag{7}$$

for all j, τ where $\mathbb{K}^{flood\, dmg.} = \{1, \cdots, K^{flood\, dmg.}\}$ and $e^{\Delta}_{jk}, e^{0}_{jk}$ define the hyperplanes.[4]

Water balance constraints ensure that the current storage is given as the previous storage plus the net inflow. The net inflow is itself defined as the sum, over all upstream reservoirs, of past releases (hm^3/day) weighted by the flow delay coefficients plus the daily inflows ξ_{jt} (hm^3/day) during time τ less the releases from all plants during the day (hm^3/day):

$$\begin{aligned} &\mathcal{S}_{j,\tau} = \mathcal{S}_{j,\tau-1}{}^{5} + \\ &\sum_{i^- \in I^-(j)} \sum_{\bar{l}=\delta^{min}_{i^-}}^{\min\{\delta^{max}_{i^-}, \tau-1\}} \lambda_{i^- \bar{l}} \mathcal{F}_{i^-, \tau-\bar{l}} - \sum_{i^+ \in I^+(j)} \mathcal{F}_{i^+, \tau+l} + \xi_{jt}, \end{aligned} \tag{8}$$

for all $j \in J, \tau \in \mathbb{T}_{t,L}$ where $\delta^{min}_{i^-}$ and $\delta^{max}_{i^-}$ represent the minimum and maximum time taken for a drop of water released from plant $i^-(j) \equiv i^-$ to reach reservoir j. We make the simplifying assumptions that $\xi_{jt} = \alpha_{jt}\xi_t$ where α_{jt} is the average proportion of inflows entering reservoir j at time t and ξ_t are the inflows over the entire catchment at time t.

As is customarily done, we model the final water value function with a piecewise linear increasing concave function [18]. More precisely, we introduce the decision variable $\bar{\mathcal{S}}_{j\tau} \geq 0$ and impose :

$$\bar{\mathcal{S}}_{j\tau} \leq s^{\Delta}_{jk}\mathcal{S}_{j\tau} + s^{0}_{jk}, \forall k \in \mathbb{K}^{f.val.}, \tag{9}$$

for all j, τ where $\mathbb{K}^{f.val.} = \{1, \cdots, K^{f.val.}\}$ and $s^{\Delta}_{jk}, s^{0}_{jk}$ define the hyperplanes, which are based on the output of other long-term scheduling tools.

The total power produced (in MW) during time τ for plant i is taken as the product of the reference production $\mathcal{P}_{i\tau}$ at some fixed reference water head times the relative water head $\mathcal{H}_{i\tau}$ [19, 20, 12]:

$$\mathcal{P}^{total}_{i\tau} = \mathcal{P}_{i\tau}\mathcal{H}_{i\tau}. \qquad \forall i \in I, \tau \in \mathbb{T}_{t,L}. \tag{10}$$

The relative water head is given by the net water head divided by the reference water head $\mathcal{H}^{ref}_i$:

$$\mathcal{H}_{i\tau} = (\mathcal{N}_{j^-\tau} - \mathcal{N}_{j^+\tau})/\mathcal{H}^{ref}_i \quad \forall i \in I, \tau \in \mathbb{T}_{t,L}. \tag{11}$$

The net water head is simply the difference between forebay $\mathcal{N}_{j^+\tau}$ and tailrace $\mathcal{N}_{j^-\tau}$ elevations during time τ, where $j^+ \equiv j^+(i)$ and $j^- \equiv j^-(i)$ respectively represent the reservoir upstream and downstream of plant i.

The forebay and tailrace elevation (m) at a given plant i are concave increasing functions of the average storage in the upstream and downstream reservoir during time τ respectively. These functions are sufficiently well approximated by affine functions of the water storage in the associated reservoir:

$$\mathcal{N}_{j^-\tau} = n^{0}_{j^-} + n^{\Delta}_{j^-}\mathcal{S}_{j^-\tau} \qquad \forall j \in J, \tau \in \mathbb{T}_{t,L} \tag{12a}$$

$$\mathcal{N}_{j^+\tau} = n^{0}_{j^+} + n^{\Delta}_{j^+}\mathcal{S}_{j^+\tau} \qquad \forall j \in J, \tau \in \mathbb{T}_{t,L}, \tag{12b}$$

[4] In reality, the impact of floods are often non-convex due to threshold effects and other complex physical phenomenon [11]. However, we believe our model is satisfactory since our policies yield very limited floods when tested on historical scenarios.

[5] $\mathcal{S}_{j,t+l}$ are already known at time t for any $l \leq 0$.

where $n_j^0, n_j^\Delta \in \mathbb{R}, \forall j$. For plants where the downstream reservoir is sufficiently far, the tailrace water level is taken as a constant $\mathcal{N}_{j^-} = \mathcal{N}_{j^+}^{ref}$.

The reference production itself depends on the total flows (hm^3/day). For every i and τ, we use the following approximation for the reference production function by using $K^{prod.\,ref.} \in \mathbb{N}$ hyperplanes:

$$\mathcal{P}_{i\tau} \leq p_{ik}^\Delta \mathcal{F}_{i\tau} + p_{ik}^0, \forall k \in \mathbb{K}^{prod.\,ref.}, \tag{13}$$

where $\mathbb{K}^{prod.\,ref.} = \{1, \cdots, K^{prod.\,ref.}\}$ and $p_{ik}^\Delta, p_{ik}^0 \in \mathbb{R}$. Each piecewise function is increasing only on a subset of its domain to reflect negative tailrace effects caused by excessive flows. Indeed, for each plant, we have $p_{ik}^\Delta > 0, k \in \mathbb{K} \setminus K^{prod.\,ref.}$ and $p_{iK^{prod.\,ref.}}^\Delta < 0$. Section 6 provides concrete examples and additional details.

3.2 Biobjective problem

We wish to find a production plan that balances some systemic measure of flood occurrence while maximizing electricity production and avoiding completely emptying the reservoirs at the end of the horizon.

Based on previous work [3], we let $\gamma_j \geq 0$ reflect the relative importance of floods at reservoir j where $\sum_{j \in J} \gamma_j = 1$. We then consider $\mathcal{O}_\tau^{floods}$, the weighted flood damages over the entire catchment at time $\tau \in \mathbb{T}_{t,L}$ (\$), which is given by the following simple relationship:

$$\mathcal{O}_\tau^{floods} = \sum_{j \in J} \gamma_j \bar{\mathcal{E}}_{j\tau}. \tag{14}$$

We evaluate the hydro-electric productive benefits associated to a reservoir release schedule at time $\tau \in \mathbb{T}_{t,L}$ by considering the (negative) total hydroelectric production value and the value of the final storage of reservoirs (\$):

$$\mathcal{O}_\tau^{prod} = \begin{cases} -\sum_{i \in I^{prod}} \mathcal{P}_{i,\tau}^{total,} \eta \nu_\tau - \sum_{j \in J} \bar{\mathcal{S}}_{j\tau} & \text{if } \tau = t + L - 1 \\ -\sum_{i \in I^{prod}} \mathcal{P}_{i,\tau}^{total} \eta \nu_\tau & \text{else,} \end{cases} \tag{15}$$

where η is the number of hours per period, ν_τ is the price of 1 MWh[6] and $\bar{\mathcal{S}}_{j\tau}$ indicates the final value of this water storage.

Finally, at the beginning of time $t \in \mathbb{T}$, we seek to minimize

$$\sum_{\tau=t}^{t+L-1} \left((1-\varphi)\mathcal{O}_\tau^{floods} + \varphi \mathcal{O}_\tau^{prod} \right), \tag{16}$$

for some $\varphi \in [0,1]$ reflecting the relative importance of the aggregate production measure $\mathcal{O}_\tau^{prod}$ compared to the aggregated flood measure $\mathcal{O}_\tau^{floods}$.

4 Incorporating uncertainty

We want to model $\{\xi_t\}_{t \in \mathbb{Z}}$, the discrete time stochastic process representing total inflows. At each time $t \in \mathbb{T} = \{1, \cdots, T\}$ for the lead times $\mathbb{L} = \{0, \cdots, L-1\}$, we assume that the $\xi \equiv \xi_{[t,t+L-1]}$ lie within the following set with probability 1:

[6] Electricity generation is assumed constant throughout each period τ. We also assume that electricity prices are known constants, which makes sense in the case of Hydro-Québec.

$$
\Xi_t{}^{8} = \left\{ \xi \in \mathbb{R}^L \;\middle|\; \begin{array}{ll} \exists \zeta, \varrho \in \mathbb{R}^L & \text{(17a)} \\ \xi_{t+l} = e^{\zeta_{t+l}}, \forall l \in \mathbb{L} & \text{(17b)} \\ \zeta = V_t \varrho + v_t & \text{(17c)} \\ W \varrho \leq w & \text{(17d)} \end{array} \right\}
$$

Representation (17) essentially states that the logarithms of inflows follow some ARIMA process, whose structure is codified by the matrix V_t and the vector v_t, and where the residuals ϱ are random variables taking values in the polyhedral set $P = \{\varrho \in \mathbb{R}^L : W\varrho \leq w\}$.

This representation captures commonly used inflow representations as a particular case. For instance, considering large P and assuming $\varrho_{[t,t+L-1]}$ follow a multivariate normal distribution implies that the inflows are correlated and approximately follow the popular log-normal distribution, often used in reservoir management problems [21, 17, 22].

The exponential function used in Equation (17b) also possesses various intuitive and statistically appealing properties. Indeed, the function ensures non-negativeness of inflows and belongs to the family of Box-Cox transforms commonly used to obtain a more adequate fit with a theoretical model [23].

5 Stochastic multi-stage formulation

We consider a dynamic setting where the inflows are progressively revealed as time unfolds over the horizon of T days. At each time $t \in \mathbb{T}$, the decision maker fixes a sequence of policies to be implemented at future times $\tau = t, \cdots, t+L-1$ under the assumptions that the future inflows $\xi_{[t,t+L-1]}$ belong to Ξ_t. Decisions must be non-anticipative in the sense that each $\mathcal{X}_\tau$ must only be a function of the past random variables $\xi_{[\tau-1]}$.

5.1 Affine decision rules

Solving the true multistage problem to optimality requires considering arbitrary functions, which will lead to an intractable problem. We therefore limit ourselves to affine policies, which are suboptimal but might still capture well the general structure of future decisions. These functions take the following form at time $\tau = t + l$ for $t \in \mathbb{T}$ and $l \in \mathbb{L}$:

$$
\mathcal{X}_{k\tau}(\xi) = \mathcal{X}^0_{k\tau} + \sum_{\tau'=t}^{t+L-1} \mathcal{X}^{\tau'}_{k\tau} \xi_{\tau'}, {}^{9} \tag{18}
$$

As mentioned in [3, 24], we replace each decision variable in the deterministic formulation by its associated affine decision in the stochastic model. To ensure that the inequalities hold with probability 1, we solve an optimization problem. For instance, constraint (1) is respected with probability 1 if $\max_{\xi \in \Xi_t} R^0_{it} + \sum_{t'=t}^{t+L-1} \mathcal{R}^{t'}_{it} \xi_{t'} \leq \bar{r}$. However, this optimization problem is intractable given our choice of Ξ_t, since this set is not generally convex for an arbitrary $t \in \mathbb{T}$ (see the Appendix A.3). Fortunately, it is possible to obtain the following exterior polyhedral conservative approximation:[10]

$$
\hat{\Xi}_t = \bigtimes_{l=1}^{L} \left[e^{\min\limits_{W\varrho \leq w} V_l^\top \varrho + v_{t,l}}, e^{\max\limits_{W\varrho \leq w} V_l^\top \varrho + v_{t,l}} \right]. \tag{19}
$$

[8] $\zeta \equiv \zeta_{[t,t+L-1]}$ and $\varrho \equiv \varrho_{[t,t+L-1]}$ represent random variables for the next future L days. We fix $V_t \in \mathbb{R}^{L \times L}$, $W \in \mathbb{R}^{c \times L}$, $v_t \in \mathbb{R}^L$ and $w \in \mathbb{R}^c$ for some $c \in \mathbb{N}$.

[9] In this case, $k \in \{n_{\tau-1} + 1, \cdots, n_{\tau-1} + n_\tau\}$ and $n_\tau \in \mathbb{N}$ represents the number of decisions at time τ. $\mathcal{X}^0_{k\tau}, \mathcal{X}^{\tau'}_{k\tau} \in \mathbb{R}, \forall k, \tau$ become the decision variables in our deterministic equivalent formulation. Non-anticipativity is respected by imposing $\mathcal{X}^{\tau'}_{k\tau} = 0, \forall \tau' \geq \tau$.

We then use strong linear programming duality to write the deterministic equivalent as a a large linear program (see [3] for details).

5.2 Objective function and composite risk

We define the (composite) risk of the production plan at the beginning of time t as:

$$\mathrm{E}\left[\sum_{l=0}^{L-1}\Big((1-\varphi)\mathcal{O}_{t+l}^{floods}(\xi)+\varphi\mathcal{O}_{t+l}^{prod}(\xi)\Big)\,|\mathcal{G}_{t-1}\right].^{11} \tag{20}$$

By linearity of the conditional expectation, it follows that this composite risk is simply a convex combination of the flood risk and the production risk. Appendix B details the analytical expression of these terms.

5.3 Using successive linear programming to approximate the true problem

Due to (10), (20) contains a bilinear term (see Appendix B for details). In order to deal with this non-convexity, we consider a commonly used first order Taylor approximation, which is detailed in the Appendix C as well as a SLP algorithm based on the general trust-region algorithm of [26]. Our algorithm begins by considering the linear problem of minimizing flood damages and builds on this solution.

The SLP algorithm displayed very rapid convergence and after only 4 iterations, the flood and production risk remained roughly constant. We obtained similar results with different stopping criteria, period of the year and weights φ in the objective function (20).

5.4 Rolling horizon simulation

We embed the SLP algorithm within a broader rolling horizon simulation that reflects the real behavior of river operators (see Figure 1).

6 The Gatineau river system

We apply our framework to the Gatineau river in Western Qubec. This hydro-electrical complex is part of the larger Outaouais catchment and is managed by Hydro-Québec. It is composed of 5 reservoirs and 3 run-of-the river plants.

The first 2 head reservoirs, Baskatong and Cabonga have storage capacity of 1563 hm^3 and 3049 hm^3, respectively, while the remaining downstream reservoirs Paugan, Chelsea and Rapides-Farmers have virtually none.

The Baskatong reservoir is the largest of the larger Outaouais-Gatineau catchment and plays a critical role in the management of the river. It is used to manage flood risk during the freshet as well as baseflow during summer. Furthermore, it is located upstream of the town of Maniwaki, which has witnessed 4 significant floods in the past and therefore imposes extremely tight operating constraints on water flows at this segment.

The three run-of-the-river plants Paugan, Chelsea and Rapides-Farmers have relatively small installed capacity. As illustrated in Figure 2, up to some critical threshold, increasing flow will increase the reference production. On the other hand, excessive releases and spills above this value will have negative effect on water head, which is captured with an additional linear segment of negative slope.

[10] Using properties of the exponential and P, we can check that $\Xi_t \subseteq \hat{\Xi}_t$. The set $\hat{\Xi}_t$ is simply the Cartesian product of intervals (*i.e.* a box) and can therefore be represented with only $2L$ hyperplanes.

[11] $\mathcal{G}_t = \sigma(\varrho_\tau : \tau \leq t)$ is the σ-algebra generated by the past ϱ_τ and that we can simply interpret as the information known at time t [25].

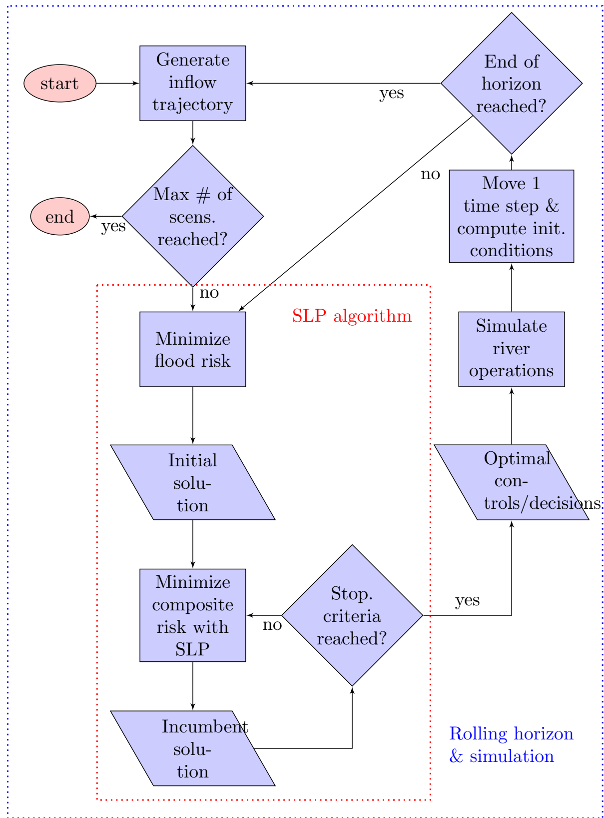

Figure 1: Flow chart for the rolling horizon simulation algorithm

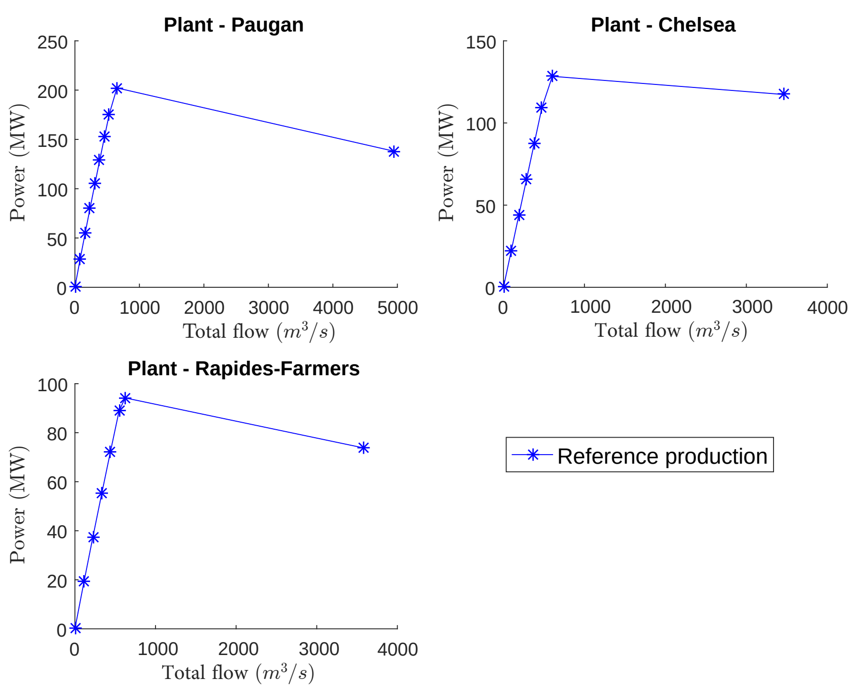

Figure 2: Production functions at reference water head

7 Numerical experiments

7.1 Daily inflows

We consider 12 years of historical daily inflows provided by Hydro-Québec. We calibrated our model on 6 years of in-sample data 1999–2004, but we also tested our framework on 6 additional out-of-sample years 2008–2013, which are on average wetter than the in-sample years.

7.2 Model performance under hard synthetic scenario compared with alternative inflow representations

The non-linear ARMA representation (17) offers significant advantages over other alternative inflow representations. This is illustrated in Table 1, which shows the results of the flood minimizing solutions[12] with three different inflow representations. Results were obtained by considering the 12 historical scenarios during the freshet under the same hard stress test where the initial storages are fixed at very high levels.

The first naive representation is a static uncertainty set that consists of intervals around the historical sample mean and does not explicitly consider serial correlation. Details can be found in [3]. The second one considers the linear ARMA(1,1) model described in [4], which is similar to (17), but does not take into account the non-linear log transformation. The third model is based on the non-linear ARMA model presented in this paper where $\xi_t = e^{\zeta_t + \hat{\mu}_t}$, $\hat{\mu}_t$ is the sample log average and the AIC criterion suggests that the ζ_t follow an AR(4) model.

Table 1: Impact of different uncertainty representations [13]

	Baskatong storage violations (hm^3)	Maniwaki flow violations (m^3/s)	Energy (GWh)	Final Volume ($10^6\, hm^3$)
Non-linear ARMA	0	0	140.44	22.22
Linear ARMA	1605.07	27.63	147.28	40.33
Naive forecast	1768.26	28.08	136.42	43.56

The absence of violations in the case of these extremely challenging stress tests is very encouraging. Moreover, these flood reduction objectives are achieved without sacrificing much energy generation compared to the best model. These results suggest that Hydro-Québec can maintain higher storages without violating operating constraints nor social obligations. However, Table 1 reveals that maintaining higher storage is of questionable value as a significant amount of water needs to be evacuated out of the system to avoid floods.

7.3 Model performance under realistic historical conditions compared with historical decisions

7.3.1 Flood reduction

Next, we consider the very wet historical scenario of fall 2003. Figure 3 illustrates that these meteorological conditions led to very high storages at the Baskatong and Cabonga reservoirs around early November and in turn to upper flow bounds violations of 55 m^3/s at the downstream town of Maniwaki on 3 occasions during the period November 14 to December 13.

Figure 3 reveals that our model out-performed the historical decisions since the production plan leads to solutions that respect all operating constraints. We are able to respect flow bounds at Maniwaki by emptying the Baskatong reservoir faster and better anticipating the very large inflows at times 5 and 6. Our release schedule maintains approximately the same final storages in Baskatong as the historical decisions,

[12] We set $\varphi = 0$ in the objective (16), which reflects the real behaviour of operators in such a critical situation.

[13] Values in the table represent the sum over 12 scenarios.

which illustrates the importance of properly timing the decisions.[14] However, as in Section 7.2, a significant amount of water is released from Cabonga out of the system.[15]

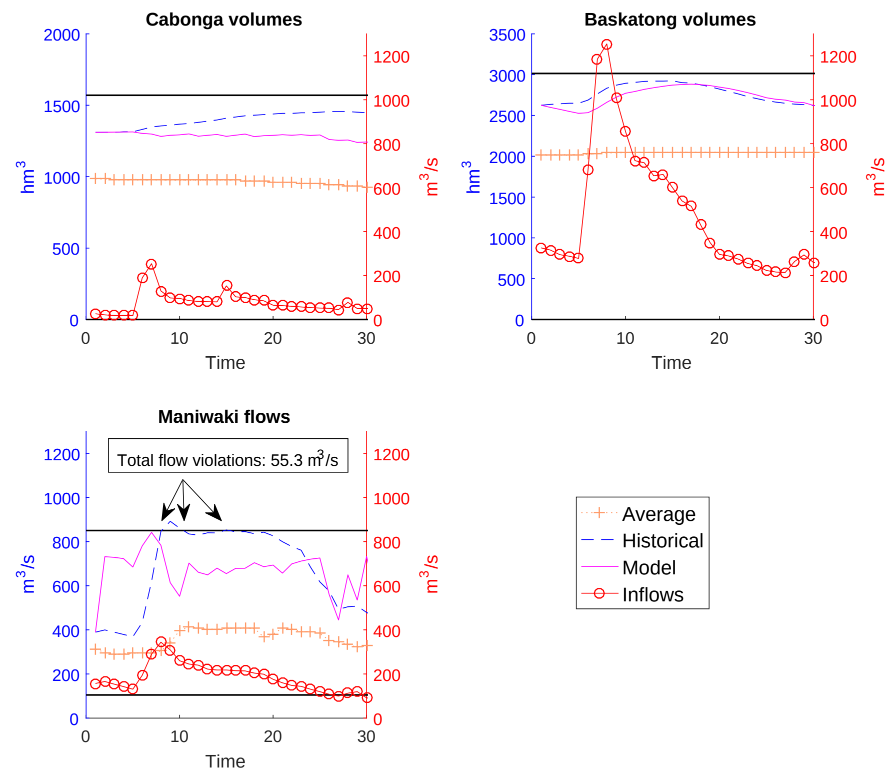

Figure 3: Model results on hard historical scenario[16]

7.3.2 Energy generation

To validate the utility of our approach for electricity generation purposes, we tested our model in the summer for the years 1999–2003.[17] We considered this period, since it represents the moment where the reservoirs are relatively full, there are no pressing flood risks and it is possible to exploit the variable water head to derive operating efficiencies. Table 2 reveals that our model yields an average energy increase of 0.3 % over a 30 day horizon relative to the historical production plan. These figures are obtained by using the same amount of water as the historical plan and avoiding spillage and floods.

[14]We consider November 14 as the starting period, which is the day the Baskatong storage reached a level very close to maximal normal operating limits. We therefore naturally fix $\varphi = 0$ and only focus on minimizing flood risk for the catchment. We fixed the initial storages at their historical levels and used the realized inflows over this 30 day period.

[15]This result suggests that under hard scenarios, it is very hard to simultaneously reduce floods and maintain the same level of storage.

[16]The time series 'Average' represents the daily average storages at Cabonga, daily average storages at Baskatong and the daily average flows at Maniwaki, respectively. These averages are computed over the 5 years 1999-2003 using the historical data. The time series 'Average', 'Historical' and 'Model' correspond to the left axis, while the right axis corresponds to the 'Inflows' time series. Upper and lower bounds are indicated by solid black lines.

[17]The summer period corresponds to the days 200–229. We only considered the years 1999–2003, because the data for the historical production plan for the years 2008–2013 was not available.

[18]Value in this table where obtained by setting $\varphi = 1$ and focusing on energy production.

Table 2: Energy generation increase (%) with model water head [18]

Year	Paugan	Chelsea	Rapides-Farmers	Total
1999	0.5	-6.5	10.1	0.2
2000	0.4	-5.4	8.6	0.4
2001	0.5	-5.2	8.0	0.4
2002	0.4	-3.5	5.4	0.2
2003	0.1	-3.5	5.1	0.0
Average	0.4	-4.8	7.4	0.3

8 Conclusion

This paper presents encouraging evidence that stochastic programming models based on affine decision rules can improve short-term river management operations. Our framework based on non-linear time series and a SLP algorithm can successfully consider important phenomenon such as water delays, variable water head and inflow persistence of arbitrary order.

More importantly, our model can yield sizable improvements when compared with historical decisions. We also show that it is possible to find feasible release schedules with storage considerably larger than average historical levels, even with high inflows. This suggests it is possible to revise the drawdown-refill cycle currently used in practice to maintain higher storage and limit unproductive spills.

Finally, the simple convex combination of flood control and production objectives we consider can allow river operators to easily explore the solution space. Although we could not give additional results due to lack of space, it is interesting to assess possible trade-offs using different inflow simulations. Since our model is extremely fast to solve, it is highly feasible to perform such sensitivity analysis in a realistic setting.

A Details on Ξ_t

A.1 Time series model linking ζ and ϱ

We assume that at each time $t \in \mathbb{Z}$, there exists some finite constant $\bar{v}_t$ such that $\zeta_t = \ln \xi_t - \bar{v}_t$[19] are zero mean random variables that satisfy the equation:

$$\phi(B)\zeta_t = \theta(B)\varrho_t, \tag{21}$$

where B represents the backshift operator acting on time indices such that $B^p\zeta_t = \zeta_{t-p}$ for all $t, p \in \mathbb{Z}$ [27, 28], $\phi(B) = 1 - \sum_{i=1}^{p} \phi_i B^i$ and $\theta(B) = 1 + \sum_{i=1}^{q} \theta_i B^i$ with $\phi_i, \theta_i \in \mathbb{R}$ for $p, q \in \mathbb{N}$.

We also suppose $\{\varrho_t\}$ are second order stationary zero-mean i.i.d. random variables [27] and that we can find $\psi(B) = \sum_{i=0}^{\infty} \psi_i B^i$ where $\phi(B)\psi(B) = \theta(B)$ such that $\sum_{i=0}^{\infty} |\psi_i| < \infty$. Under suitable conditions on $\phi(B)$ and $\theta(B)$, the representation $\zeta_t = \psi(B)\varrho_t$ holds and is essentially unique [28].

We then exploit the following linear decomposition:

$$\hat{\zeta}_t(l) \equiv \mathrm{E}\left[\zeta_{t+l} | \mathcal{G}_t\right] = \sum_{i=0}^{l-1} \psi_i \varrho_{t+l-i} \tag{22}$$

$$\rho_t(l) \equiv \zeta_{t+l} - \hat{\zeta}_t(l) = \sum_{i=0}^{l-1} \psi_i \varrho_{t+l-i}, \tag{23}$$

for any $t \in \mathbb{T}$ and $l \in \mathbb{L}$. $\hat{\zeta}_t(l)$ can be naturally interpreted as a forecast of ζ_{t+l} given information up to time t and $\rho_t(l)$ as the forecast error.

[19] Given S years of data where $\xi_{t,i}$ represents the inflows on the t^{th} period of the i^{th} series, we can namely consider $\bar{v}_t = \sum_{i=1}^{S} \frac{1}{S} \ln \xi_{t,i}, \forall t$, the sample daily logged mean, which is also used in [17].

If we set $\rho_{t-1,L} \equiv (\rho_{t-1}(1), \cdots, \rho_{t-1}(L))^\top$ for any $t \in \mathbb{Z}$, we can then express the forecast error vector $\rho_{t-1,L}$ as a linear function of the independent $\varrho_{[t,t+L-1]}$. More specifically, $\rho_{t-1,L} = V_t \varrho_{[t,t+L-1]}$ holds for all $L \in \{1, \cdots, T-t+1\}$, where $V_t \equiv V \in \mathbb{R}^{L\times L}$ is the following invertible and lower triangular square matrix, which is constant across all $t \in \mathbb{Z}$:

$$V = \begin{pmatrix} 1 & & \cdots & 0 \\ \psi_1 & 1 & & \vdots \\ \vdots & & \ddots & \\ \psi_{L-1} & \cdots & \psi_1 & 1 \end{pmatrix} \tag{24}$$

We then have the system of equalities:

$$\zeta_{[t,t+L-1]} = \hat{\zeta}_{t-1,L} + V\varrho_{[t,t+L-1]} \tag{25}$$

and it follows that $\xi_{t+l} = e^{\bar{v}_{t+l}+\hat{\zeta}_{t-1}(l+1)+\rho_{t-1}(l+1)}, \forall l \in \mathbb{L}$ and $\hat{\zeta}_{t-1,L} + \bar{v}_{[t,t+L-1]} \equiv v_t \in \mathbb{R}^L$ in representation (17) where $\hat{\zeta}_{t-1,L} \equiv (\hat{\zeta}_{t-1}(1), \cdots, \hat{\zeta}_{t-1}(L))^\top$ and $\bar{v}_{[t,t+L-1]} = (\bar{v}_t, \cdots, \bar{v}_{t+L-1})^\top$. For more details, see [4].

A.2 Polyhedral support of the ϱ_t

We fix some $\epsilon > 0$ and consider the following polyhedron:

$$P = \{\varrho \in \mathbb{R}^L : \sum_{i=1}^{L} |\varrho_i| \le L\sqrt{\epsilon\sigma_\varrho^2}; |\varrho_i| \le \sqrt{L\epsilon\sigma_\varrho^2}, \forall i\}. \tag{26}$$

This set is motivated by Markov's inequality. Indeed, for the independent random variables $\tilde{\varrho} \equiv \varrho_{[t,t+L-1]}$, we know that $\mathbb{P}(\tilde{\varrho} \in P) \ge 1 - \epsilon^{-1}$. For more details, see [4].

A.3 Counterexample showing that Ξ_t is generally non-convex

We show that in general, Ξ_t is not convex for an arbitrary $t \in \mathbb{T}$. Consider the following instance:

$$\begin{aligned} &V = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \\ &v_t = (0,0)^\top \\ &P = \{\varrho \in \mathbb{R}^L : -1 \le \varrho_l \le 1, \forall l = 1, \cdots, L; \sum_{i=1}^{L} |\varrho_i| \le \sqrt{L}\} \\ &L = 2. \end{aligned}$$

Figure 4 displays $\Xi_t = \{\xi \in \mathbb{R}^L : \exists \varrho \in P, \xi_l = \exp(V_l^\top \varrho), \forall l = 1, \cdots, L\}$ and illustrates that Ξ_t is in general not convex.

For a slightly more formal demonstration, it is possible to show that given the two points $\hat{\varrho}_1 = (1-\sqrt{2}, 1)^\top \in P$ and $\hat{\varrho}_2 = (\sqrt{2}-1, 1)^\top \in P$ illustrated in Figure 4 as well as $\lambda = \frac{1}{2}$, there exists no $\varrho \in P$ such that:

$$\lambda \begin{pmatrix} e^{V_1^\top \hat{\varrho}_1} \\ e^{V_2^\top \hat{\varrho}_1} \end{pmatrix} + (1-\lambda) \begin{pmatrix} e^{V_1^\top \hat{\varrho}_2} \\ e^{V_2^\top \hat{\varrho}_2} \end{pmatrix} = \begin{pmatrix} e^{V_1^\top \varrho} \\ e^{V_2^\top \varrho} \end{pmatrix}. \tag{27}$$

Equivalently, we can show that $\forall \varrho \in P$,

$$\left\| \begin{pmatrix} e^{V_1^\top \varrho} \\ e^{V_2^\top \varrho} \end{pmatrix} - \lambda \begin{pmatrix} e^{V_1^\top \hat{\varrho}_1} \\ e^{V_2^\top \hat{\varrho}_1} \end{pmatrix} + (1-\lambda) \begin{pmatrix} e^{V_1^\top \hat{\varrho}_2} \\ e^{V_2^\top \hat{\varrho}_2} \end{pmatrix} \right\|_\infty > 0.$$

This can be shown by solving the following linear program and observing that its optimal value is strictly larger than 0:

$$\begin{aligned} \min_{\varrho^+\geq 0, \varrho^-\geq 0, t\geq 0} \quad & t \\ \text{s. t.} \quad & \sum_{i=1}^{2}(\varrho_i^+ + \varrho_i^-) \leq \sqrt{2} \\ & \varrho_i^+ + \varrho_i^- \leq 1, \quad \forall i = 1,2 \\ & V_i^\top(\varrho^+ - \varrho^-) - l_i^\lambda \leq t, \quad i = 1,2 \\ & l_i^\lambda - V_i^\top(\varrho^+ - \varrho^-) \leq t, \quad i = 1,2, \end{aligned}$$

where $l_i^\lambda = ln(\lambda e^{V_i^\top \hat{\varrho}_1} + (1-\lambda)e^{V_i^\top \hat{\varrho}_2})$ is a known constant.

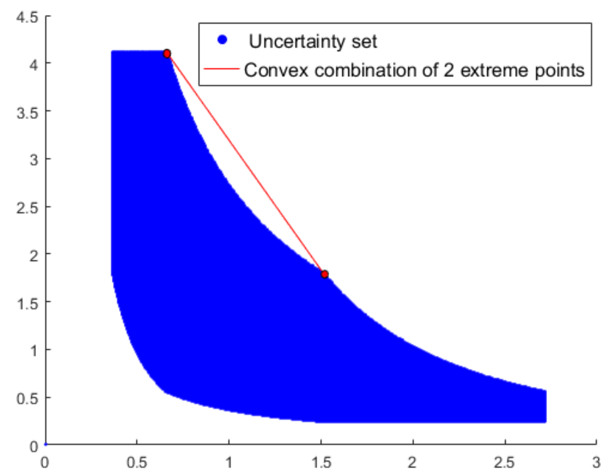

Figure 4: Non convex uncertainty set

B Analytical expression for the composite risk

B.1 Conditional expected value of $\sum_{l=0}^{L-1} \mathcal{O}_{t+l}^{floods}(\xi)$ and $\sum_{l=0}^{L-1} \mathcal{O}_{t+l}^{prod}(\xi)$

By considering affine decision rules, deriving an analytical expression for the flood and production risk becomes straightforward. Indeed,

$$\mathrm{E}\left[\sum_{l=0}^{L-1} \mathcal{O}_{t+l}^{floods}(\xi)|\mathcal{G}_{t-1}\right] = \sum_{l=0}^{L-1}\sum_{j\in J}\gamma_j \mathrm{E}\left[\bar{\mathcal{E}}_{j,t+l}(\xi)|\mathcal{G}_{t-1}\right], \tag{28}$$

where

$$\mathrm{E}\left[\bar{\mathcal{E}}_{j,t+l}(\xi)|\mathcal{G}_t\right] = \bar{\mathcal{E}}_{j,t+l}^0 + \sum_{l'=0}^{L-1}\bar{\mathcal{E}}_{j,t+l}^{t+l'}\mathrm{E}\left[\xi_{t+l'}|\mathcal{G}_t\right]. \tag{29}$$

The production risk is defined as:

$$\mathrm{E}\left[\sum_{l=0}^{L-1} \mathcal{O}_{t+l}^{prod}(\xi)|\mathcal{G}_{t-1}\right]$$

$$= - \sum_{l=0}^{L-1} \sum_{i \in I^{prod}} \mathrm{E}\left[\mathcal{P}_{i,t+l}(\xi)\mathcal{H}_{i,t+l}(\xi)|\mathcal{G}_{t-1}\right] - \sum_{j \in J} \mathrm{E}\left[\bar{\mathcal{S}}_{j,t+L-1}(\xi)|\mathcal{G}_{t-1}\right]. \quad (30)$$

The conditional expected value of the final value of storages $\mathrm{E}\left[\bar{\mathcal{S}}_{j,t+L-1}(\xi)|\mathcal{G}_{t-1}\right]$ is defined similarly to (29) while $\mathrm{E}\left[\mathcal{P}_{i,t+l}(\xi)\mathcal{H}_{i,t+l}(\xi)|\mathcal{G}_{t-1}\right]$, which involves the product of two affine functions, is defined as:

$$\begin{aligned} \mathrm{E}\left[\mathcal{P}_{i,t+l}(\xi)\mathcal{H}_{i,t+l}(\xi)|\mathcal{G}_{t-1}\right] = \quad & \mathcal{P}^0_{i,t+l}\mathcal{H}^0_{i,t+l} \\ + \sum_{k=0}^{L-1} & (\mathcal{P}^0_{i,t+l}\mathcal{H}^k_{i,t+l} + \mathcal{H}^0_{i,t+l}\mathcal{P}^k_{i,t+l})\mathrm{E}\left[\xi_{t+k}|\mathcal{G}_{t-1}\right] \\ + \sum_{m=0}^{L-1}\sum_{k=0}^{L-1} & \mathcal{P}^m_{i,t+l}\mathcal{H}^k_{i,t+l}\mathrm{E}\left[\xi_{t+m}\xi_{t+k}|\mathcal{G}_{t-1}\right]. \end{aligned} \quad (31)$$

The conditional expectations $\mathrm{E}\left[\xi_{t+k}|\mathcal{G}_{t-1}\right]$ and $\mathrm{E}\left[\xi_{t+k}\xi_{t+m}|\mathcal{G}_{t-1}\right]$ in (29) and (31) are detailed in the following Section B.2.

B.2 Conditional expected value of ξ_{t+l}

Theorem 1 *For any $t \in \mathbb{Z}$ and $l \in \mathbb{N}$:*

$$E\left[\xi_{t+l}|\mathcal{G}_t\right] = e^{\hat{\zeta}_t(l)} \cdot E\left[e^{\rho_t(l)}\right] \quad (32)$$

and for $k \geq l \geq 1$:

$$\begin{aligned} E\left[\xi_{t+l}\xi_{t+k}|\mathcal{G}_t\right] = & e^{\hat{\zeta}_t(l)+\hat{\zeta}_t(k)} \cdot E\left[e^{\sum_{i=0}^{k-l-1}\psi_i\varrho_{t+k-i}}\right] \\ & \cdot E\left[e^{\sum_{i=k-l}^{k-1}(\psi_i+\psi_{l-k+i})\varrho_{t+k-i}}\right]. \end{aligned} \quad (33)$$

Proof. This is a direct application of Theorem 34.3 of [25] together with the observation that the $\{\varrho_t\}$ are independent and hence $e^{\rho_t(l)} = e^{\sum_{i=0}^{l-1}\psi_i\varrho_{t+l-i}}$ is independent of $\{\varrho_\tau\}_{\tau=-\infty}^{t}$, which in turn implies that $\mathrm{E}\left[e^{\rho_t(l)}|\mathcal{G}_t\right] = \mathrm{E}\left[e^{\rho_t(l)}\right]$. □

At time t, $e^{\hat{\zeta}_t(l)}$ is a known deterministic value for any $l \in \mathbb{N}$ that depends on the past observed $\{\varrho_\tau\}_{\tau=-\infty}^{t}$. To compute $\mathrm{E}\left[e^{\sum_{i=0}^{k}\chi_i\varrho_{t+i}}\right]$ for any $k \in \{0,\cdots,L\}$ and $\chi_i \in \mathbb{R}, \forall i \in \{0,\cdots,L\}$, we perform Monte Carlo simulation.

C First order Taylor approximation of the composite risk

We first fix $\mathrm{E}\left[\mathcal{P}_{i,t+l}(\xi)\mathcal{H}_{i,t+l}(\xi)|\mathcal{G}_{t-1}\right] \equiv F_{i,t+l}(\mathcal{H}_{i,t+l},\mathcal{P}_{i,t+l})$ where $\mathcal{H}_{i,t+l} = (\mathcal{H}^0_{i,t+l},\mathcal{H}^{t+1}_{i,t+l},\cdots,\mathcal{H}^{t+L-1}_{i,t+l})^\top \in \mathbb{R}^L$ and $\mathcal{P}_{i,t+l} = (\mathcal{P}^0_{i,t+l},\mathcal{P}^{t+1}_{i,t+l},\cdots,\mathcal{P}^{t+L-1}_{i,t+l})^\top \in \mathbb{R}^L$. Given the point $(\hat{\mathcal{H}}^\top_{i,t+l},\hat{\mathcal{P}}^\top_{i,t+l})^\top \in \mathbb{R}^{2L}$, we then obtain:

$$
\begin{aligned}
&F_{i,t+l}(\mathcal{H}_{i,t+l}, \mathcal{P}_{i,t+l}) \\
&\quad \approx F_{i,t+l}(\hat{\mathcal{H}}_{i,t+l}, \hat{\mathcal{P}}_{i,t+l}) \\
&\qquad + \hat{\mathcal{H}}^0_{i,t+l}(\mathcal{P}^0_{i,t+l} - \hat{\mathcal{P}}^0_{i,t+l}) \\
&\qquad + \sum_{k=0}^{L-1} \hat{\mathcal{H}}^k_{i,t+l}(\mathcal{P}^0_{i,t+l} - \hat{\mathcal{P}}^0_{i,t+l})\mathrm{E}\left[\xi_{t+k}|\mathcal{G}_{t-1}\right] \\
&\qquad + \hat{\mathcal{P}}^0_{i,t+l}(\mathcal{H}^0_{i,t+l} - \hat{\mathcal{H}}^0_{i,t+l}) \\
&\qquad + \sum_{k=0}^{L-1} \hat{\mathcal{P}}^k_{i,t+l}(\mathcal{H}^0_{i,t+l} - \hat{\mathcal{H}}^0_{i,t+l})\mathrm{E}\left[\xi_{t+k}|\mathcal{G}_{t-1}\right] \\
&\qquad + \sum_{k=0}^{L-1} \hat{\mathcal{P}}^0_{i,t+l}(\mathcal{H}^{t+k}_{i,t+l} - \hat{\mathcal{H}}^{t+k}_{i,t+l})\mathrm{E}\left[\xi_{t+k}|\mathcal{G}_{t-1}\right] \\
&\qquad + \sum_{m=0}^{L-1}\sum_{k=0}^{L-1} \hat{\mathcal{P}}^{t+m}_{i,t+l}(\mathcal{H}^{t+k}_{i,t+l} - \hat{\mathcal{H}}^{t+k}_{i,t+l})\mathrm{E}\left[\xi_{t+m}\xi_{t+k}|\mathcal{G}_{t-1}\right] \\
&\qquad + \sum_{k=0}^{L-1} \hat{\mathcal{H}}^0_{i,t+l}(\mathcal{P}^{t+k}_{i,t+l} - \hat{\mathcal{P}}^{t+k}_{i,t+l})\mathrm{E}\left[\xi_{t+k}|\mathcal{G}_{t-1}\right] \\
&\qquad + \sum_{m=0}^{L-1}\sum_{k=0}^{L-1} \hat{\mathcal{H}}^{t+m}_{i,t+l}(\mathcal{P}^{t+k}_{i,t+l} - \hat{\mathcal{P}}^{t+k}_{i,t+l})\mathrm{E}\left[\xi_{t+m}\xi_{t+k}|\mathcal{G}_{t-1}\right]
\end{aligned} \tag{34}
$$

References

[1] J. Labadie, "Optimal operation of multireservoir systems: state-of-the-art review," Journal of Water Resources Planning and Management, 130, 93–111, 2004.

[2] A. Castelletti, F. Pianosi, and R. Soncini-Sessa, "Water reservoir control under economic, social and environmental constraints," Automatica, 44(6), 1595–1607, 2008.

[3] C. Gauvin, E. Delage, and M. Gendreau, "Decision rule approximations for the risk averse reservoir management problem," European Journal of Operational Research, 2017.

[4] ——, "A stochastic program with tractable time series and affine decision rules for the reservoir management problem," Les cahiers du GERAD, Tech. Rep. G-2016-24, 2016.

[5] A. Ben-Tal, E. Goryashko, A. Guslitzer, and A. Nemirovski, "Adjustable robust solutions of uncertain linear programs," Mathematical Programming, 99, 351–378, 2004.

[6] D. Kuhn, W. Wiesemann, and A. Georghiou, "Primal and dual linear decision rules in stochastic and robust optimization," Mathematical Programming Series A, 130, 177–209, 2011.

[7] A. Lorca and A. Sun, "Adaptive robust optimization with dynamic uncertainty sets for multi-period economic dispatch under significant wind," IEEE Transactions on Power Systems, 2015.

[8] A. Turgeon, "Solving a stochastic reservoir management problem with multilag autocorrelated inflows," Water Resources Research, 41, p. W12414, 2005.

[9] J. R. Stedinger and B. A. Faber, "Reservoir optimization using sampling sdp with ensemble streamflow prediction (esp) forecast," Journal of Hydrology, 249, 113–133, 2001.

[10] L. Zéphyr, P. Lang, B. F. Lamond, and P. Côté, Controlled Approximation of the Stochastic Dynamic Programming Value Function for Multi-Reservoir Systems. Cham: Springer International Publishing, 2016, 31–37.

[11] F. Pianosi and R. Soncini-Sessa, "Real-time management of a multipurpose water reservoir with a heteroscedastic inflow model," Water Resources Research, 45(10), 1–12, 2009, w10430.

[12] D. De Ladurantaye, M. Gendreau, and J.-Y. Potvin, "Optimizing profits from hydroelectricity production," Computers and Operations Research, 36, 499–529, 2009.

[13] L. Pan, M. Housh, P. Liu, X. Cai, and X. Chen, "Robust stochastic optimization for reservoir operation," Water Resources Research, 51(1), 409–429, 2015.

[14] S. Séguin, P. Côté, and C. Audet, "Short-term unit commitment and loading problem," Les cahiers du GERAD, Tech. Rep., 2014.

[15] C. Rougé and A. Tilmant, "Using stochastic dual dynamic programming in problems with multiple near-optimal solutions," Water Resources Research, 52(5), 4151–4163, 2016.

[16] M. Pereira and L. Pinto, "Multi-stage stochastic optimization applied to energy planning," Mathematical Programming, 52, 359–375, 1991.

[17] A. Shapiro, W. Tekaya, J. P. da Costa, and M. P. Soares, "Risk neutral and risk averse stochastic dual dynamic programming method," European Journal of Operational Research, 224(2), 375–391, 2013.

[18] P.-L. Carpentier, M. Gendreau, and F. Bastin, "Long-term management of a hydroelectric multireservoir system under uncertainty using the progressive hedging algorithm," Water Resources Research, 49, 2812–2827, 2013.

[19] D. De Ladurantaye, M. Gendreau, and J.-Y. Potvin, "Strategic bidding for price-taker hydroelectricity producers," IEEE Transactions on Power Systems, 22(4), 2187–2203, 2007.

[20] A. Gjelsvik, B. Mo, and A. Haugstad, Long- and Medium-term Operations Planning and Stochastic Modelling in Hydro-dominated Power Systems Based on Stochastic Dual Dynamic Programming. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, 33–55.

[21] A. Castelletti, S. Galetti, M. Restelli, and R. Soncini-Sessa, "Tree-based reinforcement learning for optimal water reservoir operation," Water Resources Research, 46, p. W09507, 2010.

[22] A. Turgeon and R. Charbonneau, "An aggregation-disaggregation approach to long-term reservoir management," Water Resources Research, 34, 3585–3594, 1998.

[23] G. E. P. Box and D. R. Cox, "An analysis of transformations," Journal of the Royal Statistical Society, 2, 211–252, 1964.

[24] R. Egging, S. E. Fleten, I. Grønvik, A. Hadziomerovic, and N. Ingvoldstad, "Linear decision rules for hydropower scheduling under uncertainty," IEEE Transactions on Power Systems, 32(1), 103–113, 2017.

[25] P. Billingsley, Probability and measure, third edition. John Wiley & Sons Inc, 1995.

[26] J. Nocedal and S. J. Wright, Numerical Optimization, ser. Springer Serie in Operations Research, P. Glynn and S. M. Robinson, Eds. Springer, 1999.

[27] G. E. P. Box, G. M. Jenkins, and G. C. Reinsel, Time series analysis: forecasting and control, 4th edition. John Wiley & Sons, Inc., Hoboken, New Jersey., 2008.

[28] P. J. Brockwell and R. A. Davis, Time series: theory and methods. Springer-Verlag New York, 1987.