

## CINFORMS ANNUAL MEETING | 2020 VIRTUAL



Affine Decision Rule Approximation to Immunize against Demand Response Uncertainty in Smart Grids' Capacity Planning

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## Outline

- i. Introduction
- ii. Generation expansion planning (GEP), ETEM
- iii. Robust ETEM
- iv. Decomposition algorithm
- v. Numerical results





## **Introduction- GEP problem**

Generation Expansion Planning (GEP) problem:

• Optimal investment plan for generation technologies

$\min_{\mathbf{x}_{t}^{i}, \mathbf{y}_{t}} \sum_{i} \rho_{t} \cdot [I_{t}(\mathbf{x}_{t}^{i}) + F_{t}]$	$(x_{i}^{t}) +$	$-V_t(y_t)]$
$\gamma_t, y_t$ t,i		Definition
(t)	$x_t^i$	Investment of technology <i>i</i> in period <i>t</i>
$g_t(x_i^t, y_t) > b_t$	$x_i^t$	Installed capacity of technology $i$ up to period $t$
$O((\gamma)) = c$	Уt	Procurement decision at period $t$
$\mathbf{x} \in \mathcal{X} \ \mathbf{y} \in \mathcal{V}$	$I_t$	Investment cost
$x \in \mathcal{R}, y \in \mathcal{Y}$	$F_t$	Fixed operational cost
	$V_t$	Variable operational cost
	$g_t(x, y_t)$	Network balance constraints

- It is a challenging problem:
  - Large scale problem
  - Presence of uncertainties





#### **Introduction - DR**

**Demand Response:** "Changes in electric usage by end-use customers in response to supply side incentives<sup>1</sup>"

DR is an alternative for expensive capacity expansion policies

Recent versions of Smart grid's GEP consider demand response (DR)

$$\min_{x_t^i, y_t, \overline{d}_t} \sum_{t,i} \rho_t \cdot [I_t(x_t^i) + F_t(x_i^t) + V_t(y_t)]$$
$$g_t(x_i^t, y_t, \overline{d}_t) \ge b_t$$
$$x \in \mathcal{X}, y \in \mathcal{Y}$$





<sup>1</sup> Energy Regulatory Commission

## **Introduction - DRU**

Integrating DR in GEP is challenging because:

- Availability of resource depends on level of contribution to DR programs;
- Level of contribution is not known at the planning phase;

Failure to address DRU leads to inappropriate installed capacity; i.e. either shortage of electricity or wasted excess capacity.

Robust Optimization (RO) and Stochastic Programming (SP) are two main tools to address DRU.





<sup>&</sup>lt;sup>1</sup> Energy Regulatory Commission

## **Introduction - RO**

- Robust Optimization (RO) is an approach to address uncertainty when the distribution of parameters is not known.
- The uncertain parameters are only known to lie within a region, called uncertainty set, and the purpose is to identify solutions that are immune with respect to realizations of the uncertain parameters.

 $\min_{x \in \mathcal{X}} \max_{z \in \mathcal{Z}} h(x, z)$ s.t.  $g_j(x, z) \ge 0, \forall z \in \mathcal{Z}, \forall j = 1, \cdots, J$ 

• Solutions obtained from RO are guaranteed to have the best performance in worst-case scenario of the uncertain parameter.





## Introduction – MPRO

Multi-period RO (MPRO) refers to problems in which decisions are taken at different points of time:

- *Here and now* decisions need to be taken before any information about uncertainty is revealed.
- *Recourse* decisions could be delayed until after observing the realizations of all uncertain parameters.
- The purpose is to make the best first-stage decision assuming the worst cost will occur for the next period.

$$\max_{x_1,\{x_t(\cdot)\}_{t=2}^T} \quad \inf_{z \in \mathcal{Z}} c_1(z)^\top x_1 + \sum_{t=2}^T c_t(z)^\top x_t(v_t(z))$$

subject to:

$$a_{j1}(z)^{ op}x_1 + \sum_{t=2}^{T} a_{jt}(z)^{T} x_t(v_t(z)) \leq b_j(z), \quad \forall z \in \mathcal{Z}, \forall j = 1, \cdots, J$$





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## **ETEM – generation expansion model**

- Energy-Technology-Environment-Model (ETEM)<sup>1</sup>
- Long term capacity expansion planning model.
  - Planning horizon: 50 years
- Large scale linear programming
- Objective: minimize total cost of capacity expansion planning:
  - Investment cost
  - Fixed cost
  - Variable cost
  - Importation cost
  - Exportation income
  - Transmission cost
- Constraints: technology constraint, network energy balance, demand response constraint, economical and environmental constraints.





<sup>&</sup>lt;sup>1</sup> F. Babonneau, M. Caramanis, A. Haurie, Etem-sg: Optimizing regional smart energy system with power distribution constraints and options, Environmental Modeling & Assessment 22 (2017) 411–4

#### **ETEM – Demand response constraint**

$$\sum_{p \in \mathbb{P}^{P_c}} \boldsymbol{P}_{t,s,l,p,c} \geq \Theta_{t,l,c} \, \boldsymbol{V}_{t,s,c} \; \forall t,s,l,c \in \mathbb{C}^{\mathcal{D}}$$

**Robust DR:** 

$$\sum_{\boldsymbol{p}\in\mathbb{P}^{P_{c}}}\boldsymbol{P}_{t,s,l,\boldsymbol{p},\boldsymbol{c}}\geq\Theta_{t,l,\boldsymbol{c}}(\boldsymbol{V}_{t,s,\boldsymbol{c}}+\boldsymbol{\delta}_{t,s,\boldsymbol{c}})\quad\forall\delta\in\Delta\quad\forall t,s,l,\boldsymbol{c}\in\mathbb{C}^{L}$$

**Budget uncertainty set:** 

$$\Delta = \left\{ \delta \in \mathbb{R}^d \left| \exists \zeta \in [-1, \, 1]^d, \, \delta_{t, s, c} = \beta_{t, s, c} \zeta_{t, s, c}, \, \forall t, s, c \in \mathbb{C}^U \right. \\ \left. \sum_{s \in \mathbb{S}^j} \sum_{c \in \mathbb{C}^U} \left| \zeta_{t, s, c} \right| \le \Gamma_{t, j} \, \forall t, j \in \mathbb{J} \right\} \right\}$$

	Definition
$\Theta_{t,l,c}$	Annual demand
$v_{t,s,c}$	Nominal DR
$\nu_{t,s,c}$	Allowed deviation from nominal DR
$\mathcal{S}_{j}$	All time-slices in season <i>j</i>
J	Set of all seasons
$\mathbb{P}^{P_c}$	Set of all technologies producing <i>c</i>
$\mathbb{P}^{C_c}$	Set of all technologies consuming <i>c</i>
$c\in\mathbb{C}^{\mathcal{D}}$	Demand
$P_{t,s,l,p,c}$	Production
$V_{t,s,c}$	Planned DR





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## **RETEM – Decision making chronology**



- First-stage decision (X)
  - Capacity expansion plan
  - Planned demand response

$$x = [\boldsymbol{C}_{t,l,p}, \boldsymbol{C}_{t,l,p}^{\mathsf{T}}, \boldsymbol{V}_{t,s,c}]^{\mathsf{T}} \ \forall t, s, l, p, c \in \mathbb{C}^{\mathcal{D}}$$

- Seasonal periodic decisions (y<sub>i</sub>)
  - Energy production
  - Energy import and export
  - Regional transmission

$$y_i = [\boldsymbol{P}_{t,s,l,p,c}, \boldsymbol{T}_{t,s,l,l' \neq l,c}, \boldsymbol{I}_{t,s,l,c}, \boldsymbol{E}_{t,s,l,c}]^\top \ \forall (t,s) \in \mathscr{A}_i, l, p, c$$



#### **RETEM – Problem formulation**

Multi-period adjustable formulation:

$$\min_{\boldsymbol{x}, \{\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}} \max_{\{\boldsymbol{\zeta}_i \in \mathcal{Z}_i\}_{i=1}^{|\mathbb{I}|}} f^\top \boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top \boldsymbol{y}_i(\boldsymbol{\zeta}_i)$$
s.t.  $A_i \boldsymbol{x} + B_i \boldsymbol{y}_i(\boldsymbol{\zeta}_i) \le b_i + C_i \boldsymbol{\zeta}_i \quad \forall \boldsymbol{\zeta}_i \in \mathcal{Z}_i(\Gamma_i), \ \forall i \in \{1...|\mathbb{I}|\}$ 
 $D \boldsymbol{x} \le e$ 

Robust multi-period conservative approximation:

$$\min_{\boldsymbol{x}, \{\boldsymbol{Y}_{i}, \boldsymbol{y}_{i}\}_{i=1}^{|\mathbb{I}|}} \max_{\{\bar{\boldsymbol{\zeta}}_{i} \in \bar{\boldsymbol{z}}_{i}\}_{i=1}^{|\mathbb{I}|}} f^{\top} \boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_{i}^{\top} \left(\boldsymbol{y}_{i} + \boldsymbol{Y}_{i} \bar{\boldsymbol{\zeta}}_{i}\right)$$

$$s.t. \quad A_{i} \boldsymbol{x} + B_{i} \left(\boldsymbol{y}_{i} + \boldsymbol{Y}_{i} \bar{\boldsymbol{\zeta}}_{i}\right) \leq b_{i} + C_{i} P_{i} \bar{\boldsymbol{\zeta}}_{i} \quad \forall \bar{\boldsymbol{\zeta}}_{i} \in \bar{\boldsymbol{\mathcal{Z}}}_{i} \; \forall i \in \{1...|\mathbb{I}|\}$$

$$D \boldsymbol{x} \leq e$$





### **RETEM – Problem formulation**

Multi-period adjustable formulation:

$$\min_{\boldsymbol{x},\{\boldsymbol{y}_i\}_{i=1}^{|||}} \max_{\{\boldsymbol{\zeta}_i \in \mathcal{Z}_i\}_{i=1}^{|||}} f^{\top} \boldsymbol{x} + \sum_{i=1}^{|||} h_i^{\top} \text{ Final reformulated problem:} \\ s.t. \quad A_i \boldsymbol{x} + B_i \boldsymbol{y}_i (\boldsymbol{\zeta}_i \\ D \boldsymbol{x} \le e \\ \textbf{x}, \{\theta_i, \Phi_i, \mathbf{Y}_i, y_i\}_{i=1}^{\top}} f^{\top} \boldsymbol{x} + \sum_{i=1}^{\tau} \nu_i^{\top} \theta_i + h_i^{\top} \boldsymbol{y}_i \\ s.t. \quad D \boldsymbol{x} \le e \\ \text{Robust multi-period conser} \\ \min_{\boldsymbol{x}, \{\boldsymbol{Y}_i, \boldsymbol{y}_i\}_{i=1}^{|||}} \max_{\{\bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i\}_{i=1}^{|||}} f^{\top} \boldsymbol{x} + \sum_{i=1}^{|||} \boldsymbol{x}_i \in \{1, ..., \tau\} \\ \min_{\boldsymbol{x}, \{\mathbf{Y}_i, \boldsymbol{y}_i\}_{i=1}^{||||}} \max_{\{\bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i\}_{i=1}^{||||}} f^{\top} \boldsymbol{x} + \sum_{i=1}^{|||} \Phi_i \nu_i \le b_i - A_i \boldsymbol{x} - B_i \boldsymbol{y}_i \quad \forall i \in \{1, ..., \tau\} \\ \Phi_i W_i = B_i \mathbf{Y}_i - C_i P_i \quad \forall i \in \{1, ..., \tau\} \\ \Phi_i, \theta_i \ge 0 \\ s.t. \quad A_i \boldsymbol{x} + B_i \left( \boldsymbol{y}_i + \boldsymbol{Y}_i \bar{\boldsymbol{\zeta}}_i \right) \le b_i + C_i P_i \boldsymbol{\zeta}_i \quad \forall \boldsymbol{\zeta}_i \in \mathcal{Z}_i \; \forall i \in \{1, ..., T\} \\ D \boldsymbol{x} \le e \\ \end{array}$$



*inferms* 

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Remember the structure of the approximation problem:

- Separable constraint (2)
- Separable second stage variable y<sub>i</sub> and Y<sub>i</sub>
- Coupling variable x and constraint (3)

#### Robust Multi Period Conservative Approximation

$$\min_{\mathbf{x}, \{\mathbf{Y}_{i}, \mathbf{y}_{i}\}_{i=1}^{|\mathbb{I}|}} \max_{\{\bar{\boldsymbol{\zeta}}_{i} \in \bar{\mathcal{Z}}_{i}\}_{i=1}^{|\mathbb{I}|}} f^{\top} \mathbf{x} + \sum_{i=1}^{|\mathbb{I}|} h_{i}^{\top} \left(\mathbf{y}_{i} + \mathbf{Y}_{i} \bar{\boldsymbol{\zeta}}_{i}\right)$$
(1)  

$$s.t. \quad A_{i} \mathbf{x} + B_{i} \left(\mathbf{y}_{i} + \mathbf{Y}_{i} \bar{\boldsymbol{\zeta}}_{i}\right) \leq b_{i} + C_{i} P_{i} \bar{\boldsymbol{\zeta}}_{i} \quad \forall \bar{\boldsymbol{\zeta}}_{i} \in \bar{\mathcal{Z}}_{i} \; \forall i \in \{1...|\mathbb{I}|\}$$
(2)  

$$D \mathbf{x} \leq e$$
(3)



Presentation of RMPCA in master-sub problems format

The (RMPCA) is equivalent to :

$$\begin{array}{ll} (\mathsf{MP}) & \min_{\mathbf{x}, \boldsymbol{\rho}, \{\mathbf{y}_i\}_{i=1}^{\tau}} f^{\top} \mathbf{x} + \sum_{i=1}^{\tau} h_i^{\top} \mathbf{y}_i + \boldsymbol{\rho}_i \\ & s.t. \quad D \mathbf{x} \leq e \\ & \boldsymbol{\rho}_i \geq g_i(\mathbf{x}, \mathbf{y}_i) \quad \forall i \in \{1..\mathbb{I}\} \\ & A_i \mathbf{x} + B_i \mathbf{y}_i \leq b_i \quad \forall i \in \{1..\mathbb{I}\} \end{array}$$

Where:

$$\begin{array}{l} (\mathsf{SP}_{i}) & \min_{\mathbf{Y}_{i}} \max_{\{\bar{\boldsymbol{\zeta}}_{i} \in \bar{\boldsymbol{\mathcal{Z}}}_{i}\}_{i=1}^{|\mathbb{I}|}} h_{i}^{\top} \mathbf{Y}_{i} \bar{\boldsymbol{\zeta}}_{i} \\ \\ s.t. & A_{i} \mathbf{x} + B_{i} \Big( \mathbf{y}_{i} + \mathbf{Y}_{i} \bar{\boldsymbol{\zeta}}_{i} \Big) \leq b_{i} + C_{i} P_{i} \bar{\boldsymbol{\zeta}}_{i} \quad \forall \bar{\boldsymbol{\zeta}}_{i} \in \bar{\boldsymbol{\mathcal{Z}}}_{i} \; \forall i \in \{1...|\mathbb{I}|\} \end{array}$$





#### Presentation of RMPCA in master-sub problems format

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$$\begin{array}{ll} \mathsf{MP}) & \min_{\mathbf{x}, \boldsymbol{\rho}, \{\mathbf{y}_i\}_{i=1}^{\tau}} f^{\top} \mathbf{x} + \sum_{i=1}^{\tau} h_i^{\top} \mathbf{y}_i + \boldsymbol{\rho}_i \\ & s.t. \quad D \mathbf{x} \leq e \\ & \boldsymbol{\rho}_i \geq g_i(\mathbf{x}, \mathbf{y}_i) \quad \forall i \in \{1..\mathbb{I}\} \\ & A_i \mathbf{x} + B_i \mathbf{y}_i \leq b_i \quad \forall i \in \{1..\mathbb{I}\} \end{array}$$

Where:

$$(\mathsf{SP}_i) \quad g_i(x, y_i) = \max_{\substack{\theta_i \ge 0, \bar{\zeta}_i \ge 0, \lambda_i \ge 0}} (-b_i + A_i x + B_i y_i)^\top \theta_i - Tr(C_i P_i \lambda_i)$$

$$s.t. \quad \theta_i \nu_i^\top - \lambda_i W_i^\top \ge 0$$

$$B_i^\top \lambda_i = -h_i \bar{\zeta}_i^\top$$

$$W_i \bar{\zeta}_i \le \nu_i$$





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$$s.t. \quad \theta_i \nu_i^\top - \lambda_i W_i^\top \ge 0$$

$$B_i^\top \lambda_i = -h_i \bar{\zeta}_i^\top$$

$$W_i \bar{\zeta}_i \le \nu_i$$





#### **Decomposition algorithm - Enhancement**

• In order to enhance MP to improve LB, replace master problem with:

$$\begin{aligned} (\mathsf{MPVI}) & \min_{\mathbf{x}, \{\boldsymbol{\rho}_i, \mathbf{y}_i\}_{i=1}^{\mathbb{I}}} f^{\top} \mathbf{x} + \sum_{i=1}^{\mathbb{I}} h_i^{\top} \mathbf{y}_i + \boldsymbol{\rho}_i \\ & s.t. \quad D \mathbf{x} \leq e \\ & A_i \mathbf{x} + B_i \mathbf{y}_i \leq b_i \quad \forall i \in \{1..\mathbb{I}\} \\ & \boldsymbol{\rho}_i \geq (-b_i + A_i \mathbf{x} + B_i \mathbf{y}_i)^{\top} \overline{\theta}_i^{\mathsf{v}} - \operatorname{Tr}(C_i P \overline{\lambda}_i^{\mathsf{v}}), \quad \forall v \in G^{\mathsf{v}}, \forall i \in \{1..\mathbb{I}\} \\ & \boldsymbol{\rho}_i \geq h_i^{\top} (\mathbf{y}_i^{l} - \mathbf{y}_i) \quad \forall l \in \Omega, \forall i \in \{1, .., \mathbb{I}\} \\ & A_i \mathbf{x} + B_i \mathbf{y}_i^{l} \leq b_i + C_i P_i \zeta_i^{l} \quad \forall l \in \Omega, \forall i \in \{1, .., \mathbb{I}\} \end{aligned}$$

• Ω is the set of last *n* worst-case scenarios identified in the algorithm.





### **Decomposition algorithm - Enhancement**

- At each iteration the master problem might have multiple solutions
- All solutions have the same worst-case performance
- But they might not have the same performance in other scenarios.
- Solve an extra optimization problem at each iteration to transmit PRO first-stage values to SPs:

$$\min_{\mathbf{x},\{\boldsymbol{\rho}_i',\boldsymbol{\rho}_i,\mathbf{y}_i\}_{i=1}^{\tau}} f^{\top} \mathbf{x} + \sum_{i=1}^{\tau} h_i^{\top} \mathbf{y}_i + \boldsymbol{\rho}_i'$$

s.t. All constraint of MPVI

$$f^{\top}\mathbf{x} + \sum_{i=1}^{\tau} h_i^{\top}\mathbf{y}_i + \rho_i \le (1+\epsilon)\mathscr{M}_{\mathbf{v}}^*$$
$$\rho_i' \ge \sum_{\mathbf{v}} \frac{1}{|G^{\mathbf{v}}|} [(-b_i + A_i\mathbf{x} + B_i\mathbf{y}_i)^{\top}\overline{\theta}_i^{\mathbf{v}} - Tr(C_iP\overline{\lambda}_i^{\mathbf{v}})] \quad \forall i \in \{1..\tau\}$$



## **Decomposition algorithm - Performance**

Table: Solution time (seconds) and number of iterations of different decomposition algorithms

$\mathbb{T}$	DET Problem-size		AARC Problem-size		BD		Р	D	PRO-BD-VI			
	Var	Con	Var Con		CPU(S)	lt	CPU(S)	lt	% impr.	CPU(S)	lt	% impr.
3	4,644	11,830	97,112	103,095	47.0	15	37.5	10	20%	26.0	4	45%
5	7,740	19,718	161,860	171,837	92.8	15	78.1	11	16%	68.9	5	26%
7	10,836	27,601	226,583	240,534	78.0	10	68.0	7	13%	48.0	3	39%
9	13,932	35,486	291,316	309,249	97.0	9	82.1	6	15%	81.3	3	16%
Average					78.7	12	66.4	8	16%	56.0	4	29%

- BD: Traditional Benders decomposition
- PRO-BD: Benders decomposition with pareto robust solutions
- RPO-BD-VI: Benders decomposition with PRO and Valid Inequalities





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#### **Case study**

- Energy system of Geneva Lake region in Switzerland <sup>3</sup>
- 140 Technologies:
  - Power-plants,
  - DGs;
  - Heat production;
  - Conventional and Flexible loads;
  - end-use transportation;
- 50 Energy commodities:
  - Resources;
  - Secondary energy;
  - final demand;
- A detailed description of final demand technologies: 100 technologies in:
  - industrial;
  - residential;
  - transportation;





<sup>&</sup>lt;sup>3</sup> F. Babonneau, M. Caramanis, A. Haurie, Etem-sg: Optimizing regional smart energy system with power distribution constraints and options, Environmental Modeling & Assessment 22 (2017) 411–4

#### **Results – Robust performance**

**DET**: deterministic model **SRM**: static robust model **RMPCA**: robust multi-period conservative approximation

To obtain average expected cost:

- Solve deterministic, AARC and Static-RC problems and store the first-stage values (*x*<sub>DET</sub>), (*x*<sub>RMPCA</sub>) and (*x*<sub>SRM</sub>) respectively.
  - Generate 1000 random scenarios for  $\zeta_i, \forall i = 1 \cdots \tau$  in the allowed deviation interval.

3

Fixing the values of  $\zeta_i$  and x to their corresponding scenario and policy, re-optimize the second-stage variables  $y_i, \forall i = 1 \cdots \tau$ .

#### Infeasibility percentage

Γ	AARC	$\operatorname{SRC}$	DET
10%	100%	100%	100%
20%	98%	96%	100%
$\mathbf{30\%}$	29%	0%	100%
40%	9%	0%	100%
50%	4%	0%	100%
60%	2%	0%	100%
70%	10%	0%	100%
80%	0%	0%	100%
90%	0%	0%	100%
100%	0%	0%	100%





#### **Results – Robust performance**

**DET**: deterministic model **SRM**: static robust model **RMPCA**: robust multi-period conservative approximation

To obtain average expected cost:

- Solve deterministic, AARC and Static-RC problems and store the first-stage values (*x*<sub>DET</sub>), (*x*<sub>RMPCA</sub>) and (*x*<sub>SRM</sub>) respectively.
- 2 Generate 1000 random scenarios for  $\zeta_i, \forall i = 1 \cdots \tau$  in the allowed deviation interval.
- Fixing the values of ζ<sub>i</sub> and x to their corresponding scenario and policy, re-optimize the second-stage variables y<sub>i</sub>, ∀i = 1 · · · τ.







#### **Results – Policy structure**



Total installed capacity by type for AARC, SRC and DET models (MW)

	2025		2030		2035		2040			2045			2050					
	RMPCA	SRM	DET															
Hydro-PP	21.3	21.3	18.9	22.4	22.4	22.4	22.4	22.4	22.4	22.6	22.6	22.6	22.8	22.8	22.8	23	23	23
PV-PP	0.2	0.2	0.2	0.2	0.2	0.2	0.8	0.8	0.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
Wind-PP	168.4	297.6	110.8	282.5	504.1	211.3	431.1	697.3	216.5	446.2	721.9	230.0	471.2	750.1	239.1	480.9	767.2	240.7
CHP	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7
Other <sup>1</sup>	0.5	0.5	0.5	1.2	0.5	0.5	1.2	3.1	0.5	1.2	3.1	0.5	1.2	3.1	0.5	3.1	3.1	3.1

<sup>1</sup> Other includes geothermal, fuel-cell and municipal-waste power plants





#### Summary

- We solved a robust multi-period conservative approximation of generation expansion model with uncertain DR.
- In the first stage the planner decides on capacity expansion strategy and planned DR.
- In seasonal stages, after observing the actual DR, the planner decides on optimal procurement of energy.
- An enhanced version of Bender's decomposition algorithm was proposed to increase the solution efficiency.
- The proposed adjustable approach can reduce the average expected cost of the system by 30% while keeping the infeasibility rate as low as 2%.
- The proposed decomposition solution method can efficiently solve realistic large-scale robust-ETEM model.







# Thank you for your attention!









#### **RETEM – Seasonal uncertainty set**

We are using a lifted uncertainty set:

$$\mathcal{Z}_i = P_i \bar{\mathcal{Z}}_i \,,$$

$$\bar{\mathcal{Z}}_i = \left\{ \bar{\zeta}_i \in R^{r_i} \,\middle| \, W_i \bar{\zeta}_i \le \nu_i, \bar{\zeta}_i \ge 0 \right\}$$

With true definition of matrices:

$$W_i := \begin{bmatrix} I_{d_i} & I_{d_i} \\ \mathbf{1}_{1 \times d_i} & \mathbf{1}_{1 \times d_i} \end{bmatrix} \qquad \qquad \nu_i := \begin{bmatrix} \mathbf{1}_{d_i \times 1} \\ \Gamma_i \end{bmatrix} \qquad \qquad P_i := \begin{bmatrix} I_{d_i} & -I_{d_i} \end{bmatrix}$$





#### **Results – Robust performance**

DET: deterministic model SRC: static robust counterpart AARC: affinely adjustable robust counterpart To obtain average expected cost:

- Solve deterministic, AARC and Static-RC problems and store the first-stage values ( $x_{DET}$ ), ( $x_{AARC}$ ) and ( $x_{SRC}$ ) respectively.
- Generate 1000 random scenarios for ζ<sub>i</sub>, ∀i = 1 · · · τ in the allowed deviation interval.
- **3** Fixing the values of  $\zeta_i$  and x to their corresponding scenario and policy, re-optimize the second-stage variables  $y_i, \forall i = 1 \cdots \tau$ .





