Worst-case regret minimization in a two-stage linear program

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International Conference on Stochastic Programming
Trondheim, August 1st, 2019

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Illustrative example

- Consider a newsvendor problem:

  Sales price : 1.25$/unit

  Ordering cost : 1$/unit

  Demand : [0, 100] units

Robust solution → 0 unit

Minmax Regret solution → 20 units

Order\Demand	0 unit	50 units	100 units	Worst-case profit	Worst-case regret
0 unit	0 $	0 $	0 $	0 $	25 $
20 units	-20 $	5 $	5 $	-20 $	20 $
100 units	-100 $	-37,50 $	25 $	-100 $	100 $

Two-stage linear program

We consider the following two-stage linear program:

$$\underset{x \in \mathcal{X}}{\text{maximize}} \quad \Big\{ h(x, \boldsymbol{\zeta}) \Big\}_{\boldsymbol{\zeta} \in \mathcal{U}}$$

with $\mathcal{X} := \{\boldsymbol{x} \in \mathbb{R}^{n_x} \,|\, W\boldsymbol{x} \leq \boldsymbol{v}\}$, $\quad \mathcal{U} := \{\boldsymbol{\zeta} \in \mathbb{R}^{n_\zeta} \,|\, P\boldsymbol{\zeta} \leq \boldsymbol{q}\}$

and either RHS uncertainty:

$$h(\boldsymbol{x}, \boldsymbol{\zeta}) := \quad \max_{\boldsymbol{y}} \; \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y}$$
$$\text{subject to } \; A\boldsymbol{x} + B\boldsymbol{y} \leq \Psi\boldsymbol{\zeta} + \boldsymbol{\psi}$$

or objective uncertainty:

$$h(\boldsymbol{x}, \boldsymbol{\zeta}) := \quad \max_{\boldsymbol{y}} \; \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}(\boldsymbol{\zeta})^T\boldsymbol{y}$$
$$\text{subject to } \; A\boldsymbol{x} + B\boldsymbol{y} \leq \boldsymbol{\psi}$$

Robust vs. Regret formulations

- Robust optimization (RO):

$$\underset{\boldsymbol{x}\in\mathcal{X}}{\text{maximize}} \quad \min_{\boldsymbol{\zeta}\in\mathcal{U}} h(x,\boldsymbol{\zeta})$$

- Worst-case Absolute Regret Minimization:

$$\text{(WCARM)} \quad \underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \;\; \max_{\boldsymbol{\zeta}\in\mathcal{U}} \;\; \left\{ \max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta}) - h(\boldsymbol{x},\boldsymbol{\zeta}) \right\}$$

- Worst-case Relative Regret Minimization:

$$\text{(WCRRM)} \quad \underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \;\; \max_{\boldsymbol{\zeta}\in\mathcal{U}} \;\; \left\{ \frac{\max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta}) - h(\boldsymbol{x},\boldsymbol{\zeta})}{\max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta})} \right\}$$

Literature on Minimax Regret Minimization

- Definition and axiomatization (Savage [1951], Milnor [1954],Stoye [2011])
- Empirical evidence of « regret aversion » (Bleichrodt et al. [2010], G. Loomes & R. Sugden (1982))
- « Less conservative than RO » (Savage [1951], Perakis and Roels [2008]; Natarajan et al. [2014],…)
- Difficulty of resolution:
  - Many combinatorial problems are NP-hard (Aissi et al. [2009])
  - Some single-stage LP problems are known to be solvable in polynomial time
    - RHS & box uncertainty (Gabrel & Murat [2010])
    - Resource allocation with objective and box uncertainty (Averbakh [2004])
  - General single-stage LP with obj. and box uncertainty is NP-hard (Averbakh & Lebedev [2005])
- Solution schemes mostly rely on Vertex Enumeration + Decomposition with MILP slave problem (Inuiguchi & Sakawa [1995], Ng [2013], Ning & You [2018])
- Many applications to production scheduling, portfolio selection, knapsack problems, pricing, newsvendor, …

Outline

- Introduction
- Regret minimization is as difficult as Two-Stage Robust Optimization
- Optimality of Affine Decision Rules
- Numerical experiments
- Conclusion

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Regret minimization is as difficult as two-stage robust optimization

Proposition. *Under reasonable conditions, both the WCARM and WCRRM problems with either objective or right-hand-side uncertainty can be equivalently reformulated using the following two-stage linear robust optimization model:*

$$
\begin{array}{lll}
(TSLRO) & \underset{\boldsymbol{x},\boldsymbol{y}(\cdot)}{\text{maximize}} & \min_{\boldsymbol{\zeta}\in\mathcal{U}} \; (C\boldsymbol{\zeta}+\boldsymbol{c})^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y}(\boldsymbol{\zeta}) + \boldsymbol{f}^T\boldsymbol{\zeta} \\
& \text{subject to} & A\boldsymbol{x} + B\boldsymbol{y}(\boldsymbol{\zeta}) \leq \Psi(\boldsymbol{x})\boldsymbol{\zeta} + \boldsymbol{\psi}\,,\ \forall\,\boldsymbol{\zeta}\in\mathcal{U} \\
& & \boldsymbol{x}\in\mathcal{X}
\end{array}
$$

TSLRO solution schemes can be ported to regret problems

- Exact methods (Zeng & Zhao [2013], Ayoub & Poss [2016])
- Conservative approximations using:
  - Linear decision rules (Ben-Tal et al. [2004], Kuhn et al. [2011])
  - Non-linear decision rules (Chen et al. [2008], Bertsimas et al. [2011])
  - Finite adaptability (Bertsimas & Caramanis, [2010])
  - Adaptive partitioning (Bertsimas & Dunning [2016], Postek & den Hertog [2016])
- Dualized reformulations & finite scenario approach (Bertsimas & de Ruiter [2016])
- Fourier-Motzkin elimination (Zhen et al. [2018])
- Copositive programming reformulations (Xu & Burer [2018], Hanasusanto & Kuhn [2018])

*See recent survey (Yanikoglu et al. [2018])

Assumptions for reformulating WCARM

	WCARM	WCRRM
Obj.	?	…
RHS	?	…

1. The set $\mathcal{X}$ is a non-empty polyhedron

2. The problem satisfies the property of relatively complete recourse

$$\forall \boldsymbol{x} \in \mathcal{X},\ \forall \boldsymbol{\zeta} \in \mathcal{U},\ \exists \boldsymbol{y} \in \mathbb{R}^{n_y}, A\boldsymbol{x} + B\boldsymbol{y} \leq \Psi(\boldsymbol{x})\boldsymbol{\zeta} + \boldsymbol{\psi}$$

WCARM with RHS uncertainty

	WCARM	WCRRM
Obj.	?	...
RHS	**TSLRO** ?	...

$$\text{WCARM} \equiv \underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \quad \max_{\boldsymbol{\zeta}\in\mathcal{U}} \left\{ \overbrace{\max_{\boldsymbol{x}'\in\mathcal{X},\boldsymbol{y}'\in\mathcal{Y}(\boldsymbol{x}',\boldsymbol{\zeta})} \boldsymbol{c}^T\boldsymbol{x}' + \boldsymbol{d}^T\boldsymbol{y}'}^{\text{best profit reachable}} - \overbrace{\max_{\boldsymbol{y}\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\zeta})} \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y}}^{\text{best recourse}} \right\}$$

$$\equiv \underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \quad \max_{\boldsymbol{\zeta}\in\mathcal{U},\boldsymbol{x}'\in\mathcal{X},\boldsymbol{y}'\in\mathcal{Y}(\boldsymbol{x}',\boldsymbol{\zeta})} \quad \min_{\boldsymbol{y}\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\zeta})} \boldsymbol{c}^T\boldsymbol{x}' + \boldsymbol{d}^T\boldsymbol{y}' - \boldsymbol{c}^T\boldsymbol{x} - \boldsymbol{d}^T\boldsymbol{y}$$

$$\equiv \underset{\boldsymbol{x}\in\mathcal{X}}{\text{maximize}} \quad \min_{\boldsymbol{\zeta}\in\mathcal{U},\boldsymbol{x}'\in\mathcal{X},\boldsymbol{y}'\in\mathcal{Y}(\boldsymbol{x}',\boldsymbol{\zeta})} \quad \max_{\boldsymbol{y}\in\mathcal{Y}(\boldsymbol{x},\boldsymbol{\zeta})} -\boldsymbol{c}^T\boldsymbol{x}' - \boldsymbol{d}^T\boldsymbol{y}' + \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y}$$

≡ TSLRO !!!

where

$$\mathcal{Y}(\boldsymbol{x},\boldsymbol{\zeta}) := \{\boldsymbol{y} \in \mathbb{R}^{n_y} | A\boldsymbol{x} + B\boldsymbol{y} \le \Psi\boldsymbol{\zeta} + \boldsymbol{\psi}\}$$

WCARM with objective uncertainty

	WCARM	WCRRM
Obj.	**?**	...
RHS	TSLRO	...

$$h(\boldsymbol{x},\boldsymbol{\zeta}) = \max_{\boldsymbol{y}} \; \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T(\boldsymbol{\zeta})\boldsymbol{y} \quad \text{subject to} \; A\boldsymbol{x} + B\boldsymbol{y} \leq \boldsymbol{\psi} \quad = \quad \min_{\boldsymbol{\rho}} \; \boldsymbol{c}^T\boldsymbol{x} + (\boldsymbol{\psi} - A\boldsymbol{x})^T\boldsymbol{\rho} \quad \text{subject to} \; B^T\boldsymbol{\rho} = \boldsymbol{d}(\boldsymbol{\zeta}), \; \boldsymbol{\rho} \geq 0$$

$$\max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta}) = \max_{\boldsymbol{x}',\boldsymbol{y}'} \; \boldsymbol{c}^T\boldsymbol{x}' + \boldsymbol{d}^T(\boldsymbol{\zeta})\boldsymbol{y}' \quad \text{subject to} \; A\boldsymbol{x}' + B\boldsymbol{y}' \leq \boldsymbol{\psi}, \; W\boldsymbol{x}' \leq \boldsymbol{v} \quad = \quad \min_{\boldsymbol{\lambda},\boldsymbol{\gamma}} \; \boldsymbol{\psi}^T\boldsymbol{\lambda} + \boldsymbol{v}^T\boldsymbol{\gamma} \quad \text{subject to} \; A^T\boldsymbol{\lambda} + W^T\boldsymbol{\gamma} = \boldsymbol{c}, \; B^T\boldsymbol{\lambda} = \boldsymbol{d}(\boldsymbol{\zeta})\,. \; \boldsymbol{\lambda}\geq 0, \boldsymbol{\gamma}\geq 0$$

WCARM with objective uncertainty

	WCARM	WCRRM
Obj.	**?**	…
RHS	TSLRO	…

$$h(\boldsymbol{x},\boldsymbol{\zeta}) = \max_{\boldsymbol{y}} \ \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T(\boldsymbol{\zeta})\boldsymbol{y} \quad \text{subject to} \quad A\boldsymbol{x} + B\boldsymbol{y} \le \boldsymbol{\psi} \quad = \quad \min_{\boldsymbol{\rho}} \ \boldsymbol{c}^T\boldsymbol{x} + (\boldsymbol{\psi} - A\boldsymbol{x})^T\boldsymbol{\rho} \quad \text{subject to} \quad B^T\boldsymbol{\rho} = \boldsymbol{d}(\boldsymbol{\zeta}), \ \boldsymbol{\rho} \ge 0$$

$$\max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta}) = \max_{\boldsymbol{x}',\boldsymbol{y}'} \ \boldsymbol{c}^T\boldsymbol{x}' + \boldsymbol{d}^T(\boldsymbol{\zeta})\boldsymbol{y}' \quad \text{subject to} \quad A\boldsymbol{x}' + B\boldsymbol{y}' \le \boldsymbol{\psi}, \ W\boldsymbol{x}' \le \boldsymbol{v} \quad = \quad \min_{\boldsymbol{\lambda},\boldsymbol{\gamma}} \ \boldsymbol{\psi}^T\boldsymbol{\lambda} + \boldsymbol{v}^T\boldsymbol{\gamma} \quad \text{subject to} \quad A^T\boldsymbol{\lambda} + W^T\boldsymbol{\gamma} = \boldsymbol{c}, \ B^T\boldsymbol{\lambda} = \boldsymbol{d}(\boldsymbol{\zeta}), \ \boldsymbol{\lambda}\ge 0, \boldsymbol{\gamma}\ge 0$$

WCARM with objective uncertainty

	WCARM	WCRRM
Obj.	**?**	…
RHS	TSLRO	…

$$h(\boldsymbol{x},\boldsymbol{\zeta}) = \min_{\boldsymbol{\rho}} \quad \boldsymbol{c}^T\boldsymbol{x} + (\boldsymbol{\psi} - A\boldsymbol{x})^T\boldsymbol{\rho}$$
$$\text{subject to} \quad B^T\boldsymbol{\rho} = \boldsymbol{d}(\boldsymbol{\zeta})$$
$$\boldsymbol{\rho} \geq 0$$

$$\max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta}) = \min_{\boldsymbol{\lambda},\boldsymbol{\gamma}} \quad \boldsymbol{\psi}^T\boldsymbol{\lambda} + \boldsymbol{v}^T\boldsymbol{\gamma}$$
$$\text{subject to} \quad A^T\boldsymbol{\lambda} + W^T\boldsymbol{\gamma} = \boldsymbol{c}$$
$$B^T\boldsymbol{\lambda} = \boldsymbol{d}(\boldsymbol{\zeta})\,.$$
$$\boldsymbol{\lambda}\geq 0, \boldsymbol{\gamma}\geq 0$$

WCARM with objective uncertainty

	WCARM	WCRRM
Obj.	**?**	...
RHS	TSLRO	...

$$
\begin{aligned}
h(\boldsymbol{x}, \boldsymbol{\zeta}) = \min_{\boldsymbol{\rho}} \quad & \boldsymbol{c}^T \boldsymbol{x} + (\boldsymbol{\psi} - A\boldsymbol{x})^T \boldsymbol{\rho} \\
\text{subject to} \quad & B^T \boldsymbol{\rho} = \boldsymbol{d}(\boldsymbol{\zeta}) \\
& \boldsymbol{\rho} \geq 0
\end{aligned}
$$

$$
\begin{aligned}
\max_{\boldsymbol{x}' \in \mathcal{X}} h(\boldsymbol{x}', \boldsymbol{\zeta}) = \min_{\boldsymbol{\lambda}, \boldsymbol{\gamma}} \quad & \boldsymbol{\psi}^T \boldsymbol{\lambda} + \boldsymbol{v}^T \boldsymbol{\gamma} \\
\text{subject to} \quad & A^T \boldsymbol{\lambda} + W^T \boldsymbol{\gamma} = \boldsymbol{c} \\
& B^T \boldsymbol{\lambda} = \boldsymbol{d}(\boldsymbol{\zeta})\,. \\
& \boldsymbol{\lambda} \geq 0, \boldsymbol{\gamma} \geq 0
\end{aligned}
$$

WCARM with objective uncertainty

	WCARM	WCRRM
Obj.	TSLRO	…
RHS	TSLRO	…

$$\max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta}) = \min_{\boldsymbol{\lambda},\boldsymbol{\gamma}} \ \boldsymbol{\psi}^T\boldsymbol{\lambda} + \boldsymbol{v}^T\boldsymbol{\gamma}$$

subject to $\Upsilon_1(\boldsymbol{\zeta})$: $A^T\boldsymbol{\lambda} + W^T\boldsymbol{\gamma} = \boldsymbol{c}$, $B^T\boldsymbol{\lambda} = \boldsymbol{d}(\boldsymbol{\zeta})$, $\boldsymbol{\lambda}\geq 0, \boldsymbol{\gamma}\geq 0$

$$h(\boldsymbol{x},\boldsymbol{\zeta}) = \min_{\boldsymbol{\rho}} \ \boldsymbol{c}^T\boldsymbol{x} + (\boldsymbol{\psi} - A\boldsymbol{x})^T\boldsymbol{\rho}$$

subject to $\Upsilon_2(\boldsymbol{\zeta})$: $B^T\boldsymbol{\rho} = \boldsymbol{d}(\boldsymbol{\zeta})$, $\boldsymbol{\rho}\geq 0$

$$\text{WCARM} \equiv \underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \ \max_{\boldsymbol{\zeta}\in\mathcal{U}} \left\{ \max_{\boldsymbol{x}'\in\mathcal{X}} h(\boldsymbol{x}',\boldsymbol{\zeta}) - h(\boldsymbol{x},\boldsymbol{\zeta}) \right\}$$

$$\equiv \underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \ \max_{\boldsymbol{\zeta}\in\mathcal{U}} \left\{ \min_{(\boldsymbol{\lambda},\boldsymbol{\gamma})\in\Upsilon_1(\boldsymbol{\zeta})} \boldsymbol{\psi}^T\boldsymbol{\lambda} + \boldsymbol{v}^T\boldsymbol{\gamma} - \min_{\boldsymbol{\rho}\in\Upsilon_2(\boldsymbol{\zeta})} \boldsymbol{c}^T\boldsymbol{x} + (\boldsymbol{\psi} - A\boldsymbol{x})^T\boldsymbol{\rho} \right\}$$

$$\equiv \underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \ \max_{\boldsymbol{\zeta}\in\mathcal{U},\boldsymbol{\rho}\in\Upsilon_2(\boldsymbol{\zeta})} \ \min_{(\boldsymbol{\lambda},\boldsymbol{\gamma})\in\Upsilon_1(\boldsymbol{\zeta})} \ \boldsymbol{\psi}^T\boldsymbol{\lambda} + \boldsymbol{v}^T\boldsymbol{\gamma} - \boldsymbol{c}^T\boldsymbol{x} - (\boldsymbol{\psi} - A\boldsymbol{x})^T\boldsymbol{\rho}$$

$$\equiv \underset{\boldsymbol{x}\in\mathcal{X}}{\text{maximize}} \ \min_{\boldsymbol{\zeta}\in\mathcal{U},\boldsymbol{\rho}\in\Upsilon_2(\boldsymbol{\zeta})} \ \max_{(\boldsymbol{\lambda},\boldsymbol{\gamma})\in\Upsilon_1(\boldsymbol{\zeta})} \ -\boldsymbol{\psi}^T\boldsymbol{\lambda} - \boldsymbol{v}^T\boldsymbol{\gamma} + \boldsymbol{c}^T\boldsymbol{x} + (\boldsymbol{\psi} - A\boldsymbol{x})^T\boldsymbol{\rho}$$

≡ TSLRO !!!

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Conservative Approximation using Affine Decision Rules

- It is common to approximate a TSLRO problem by focusing on the class of affine recourse policies

$$(AARC) \quad \underset{\boldsymbol{x},\boldsymbol{y},Y}{\text{maximize}} \quad \min_{\boldsymbol{\zeta}\in\mathcal{U}} \; (C\boldsymbol{\zeta}+\boldsymbol{c})^T\boldsymbol{x} + \boldsymbol{d}^T(\boldsymbol{y}+Y\boldsymbol{\zeta}) + \boldsymbol{f}^T\boldsymbol{\zeta}$$

$$\text{subject to} \quad A\boldsymbol{x} + B(\boldsymbol{y}+Y\boldsymbol{\zeta}) \leq \Psi(\boldsymbol{x})\boldsymbol{\zeta} + \boldsymbol{\psi}\,, \; \forall\,\boldsymbol{\zeta}\in\mathcal{U}$$

$$\boldsymbol{x}\in\mathcal{X}$$

- This gives rise to the following linear program:

$$\underset{\boldsymbol{x}\in\mathcal{X},\boldsymbol{y},Y,\Lambda,\boldsymbol{\lambda}}{\text{maximize}} \quad \boldsymbol{c}^T\boldsymbol{x} + \boldsymbol{d}^T\boldsymbol{y} - \boldsymbol{q}^T\boldsymbol{\lambda}$$

$$\text{subject to} \quad C^T\boldsymbol{x} + Y^T\boldsymbol{d} + \boldsymbol{f} + P^T\boldsymbol{\lambda} = 0$$

$$A\boldsymbol{x} + B\boldsymbol{y} - \boldsymbol{\psi} + \Lambda\boldsymbol{q} \leq 0$$

$$\Psi(\boldsymbol{x}) - BY + \Lambda P = 0$$

$$\Lambda \geq 0, \boldsymbol{\lambda} \geq 0$$

Optimality of affine decision rules (I)

Proposition. *If* $\mathcal{Z} := \{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \,|\, \boldsymbol{x} \in \mathcal{X}, A\boldsymbol{x} + B\boldsymbol{y} \leq \boldsymbol{\psi}\}$ *is a simplex set, then AARC is exact for the TSLRO reformulation of the WCARM and WCRRM problems with objective uncertainty.*

- Provides a tractable reformulation for the WCARM and WCRRM versions of the resource allocation problem

$$\underset{\boldsymbol{x}\in\mathcal{X}}{\text{minimize}} \; \sup_{\boldsymbol{\zeta}\in\mathcal{U}} \; (\max_{\boldsymbol{x}'\in\mathcal{X}} \boldsymbol{d}(\boldsymbol{\zeta})^T\boldsymbol{x} - \boldsymbol{d}(\boldsymbol{\zeta})^T\boldsymbol{x})$$

where

$$\mathcal{X} := \{\boldsymbol{x} \in \mathbb{R}^{n_x}_+ \,|\, \boldsymbol{w}^T\boldsymbol{x} \leq v\}$$

Optimality of affine decision rules (II)

Proposition. *If* $h(\boldsymbol{x},\boldsymbol{\zeta})$ *satisfies* $\max_{\boldsymbol{x}\in\mathcal{X}} h(\boldsymbol{x},\boldsymbol{\zeta}) = \boldsymbol{\gamma}^T\boldsymbol{\zeta} + \bar{\gamma}$ *and* $\mathcal{U}$ *is a simplex set, then AARC is exact for the TSLRO reformulation of the WCARM and WCRRM problems with right-hand side uncertainty.*

- Applies for the uncapacitated multi-item newsvendor problem :

$$\max_{\boldsymbol{x}\geq 0} h(\boldsymbol{x},\boldsymbol{\zeta}) := \sum_{i=1}^{n_y} (p_i - c_i)x_i + \min(-b_i(\zeta_i - x_i), (s_i - p_i)(x_i - \zeta_i))$$
$$= (\boldsymbol{p} - \boldsymbol{c})^T\boldsymbol{\zeta}$$

- Applies for the uncapacitated lot sizing problem with backlog:

$$\max_{\boldsymbol{x}\geq 0} \sum_{t=1}^{T} -c_t x_t - \max\left(h_t(\sum_{t'=1}^{t} x_{t'} - \zeta_{t'}), b_t(\sum_{t'=1}^{t} \zeta_{t'} - x_{t'})\right)$$

Optimality of affine decision rules (III)

Proposition. *If $h(\boldsymbol{x}, \boldsymbol{\zeta})$ is a sum of of piecewise linear concave functions, $\mathcal{U}$ is the budgeted uncertainty set, and the following conditions are satisfied:*

1. *Either of the following applies:*
   - $\Gamma = 1$
   - $\Gamma = m$ *and uncertainty is "additive"*
   - Γ *is integer and objective is "decomposable"*

   Conditions from Ardestani-Jaafari & D [2016]

2. $\max_{\boldsymbol{x} \in \mathcal{X}} h(\boldsymbol{x}, \boldsymbol{\zeta}) = \boldsymbol{\gamma}^T \boldsymbol{\zeta} + \bar{\gamma}$

Then AARC is exact for the TSLRO reformulation of WCARM.

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Multi-item newsvendor problem

$$\max_{\boldsymbol{x}\geq 0} \inf_{\boldsymbol{\zeta}\in\mathcal{U}} \sum_{i=1}^{n_y} (p_i - c_i)x_i + \min(-b_i(\zeta_i - x_i), (s_i - p_i)(x_i - \zeta_i))$$

- Uncorrelated demand : $\zeta \in \left\{ \zeta \,\middle|\, \begin{array}{c} \exists \delta,\ \|\delta\|_\infty \leq 1,\ \|\delta\|_1 \leq \Gamma \\ \zeta_i = \bar{\zeta}_i + \hat{\zeta}_i \delta_i \end{array} \right\}$

Newsvendor Problem Uncorrelated Demand			Gamma 30%	 50%	 70%	 100%
10 items	Worst-case Profit (RO)	Avg Gap (%) - AARC	0,00%	0,00%	0,00%	0,00%
		Avg CPU time (s) - AARC	1,42	1,43	1,45	1,47
		Avg CPU time (s) - C&CG	96,88	138,50	282,63	174,82
	Worst-case Absolute Regret (WCARM)	Avg Gap (%) - AARC	0,00%	0,00%	0,00%	0,00%
		Avg CPU time (s) - AARC	1,45	1,47	1,48	1,50
		Avg CPU time (s) - C&CG	183,95	239,36	201,82	153,05
	Worst-case Relative Regret (WCRRM)	Avg Absolute Gap (%) - AARC	0,00%	0,00%	0,00%	0,00%
		Avg CPU time (s) - AARC	0,36	0,38	0,41	0,42
		Avg CPU time (s) - C&CG	238,46	315,04	312,62	206,17
20 items	Worst-case Profit (RO)	Avg Gap (%) - AARC	0,00%	0,00%	0,00%	0,00%
		Avg CPU time (s) - AARC	1,87	1,93	2,01	2,06
		Avg CPU time (s) - C&CG	227,32	381,86	649,82	460,30
	Worst-case Absolute Regret (WCARM)	Avg Gap (%) - AARC	0,00%	0,00%	0,00%	0,00%
		Avg CPU time (s) - AARC	2,04	2,13	2,21	2,23
		Avg CPU time (s) - C&CG	494,66	760,61	781,34	367,71
	Worst-case Relative Regret (WCRRM)	Avg Absolute Gap (%) - AARC	0,00%	0,00%	0,00%	0,00%
		Avg CPU time (s) - AARC	0,98	1,05	1,15	1,26
		Avg CPU time (s) - C&CG	891,13	> 10 276,80	>14400,00	5 115,30

Avg denotes the average of 10 randomly generated instances

Multi-item newsvendor problem

$$\max_{\boldsymbol{x}\geq 0} \inf_{\boldsymbol{\zeta}\in\mathcal{U}} \sum_{i=1}^{n_y} (p_i - c_i)x_i + \min(-b_i(\zeta_i - x_i), (s_i - p_i)(x_i - \zeta_i))$$

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- Uncorrelated demand : $\zeta \in \left\{ \zeta \middle| \begin{array}{c} \exists \delta,\ \|\delta\|_\infty \leq 1,\ \|\delta\|_1 \leq \Gamma \\ \zeta_i = \bar{\zeta}_i + \hat{\zeta}_i \delta_i \end{array} \right\}$

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		Avg CPU time (s) - AARC	0,36	0,38	0,41	0,42
		Avg CPU time (s) - C&CG	238,46	315,04	312,62	206,17
20 items	Worst-case Profit (RO)	Avg Gap (%) - AARC	**0,00%**	**0,00%**	**0,00%**	**0,00%**
		Avg CPU time (s) - AARC	1,87	1,93	2,01	2,06
		Avg CPU time (s) - C&CG	227,32	381,86	649,82	460,30
	Worst-case Absolute Regret (WCARM)	Avg Gap (%) - AARC	**0,00%**	**0,00%**	**0,00%**	**0,00%**
		Avg CPU time (s) - AARC	2,04	2,13	2,21	2,23
		Avg CPU time (s) - C&CG	494,66	760,61	781,34	367,71
	Worst-case Relative Regret (WCRRM)	Avg Absolute Gap (%) - AARC	**0,00%**	**0,00%**	**0,00%**	**0,00%**
		Avg CPU time (s) - AARC	0,98	1,05	1,15	1,26
		Avg CPU time (s) - C&CG	891,13	> 10 276,80	>14400,00	5 115,30

Avg denotes the average of 10 randomly generated instances

Multi-item newsvendor problem

$$\max_{\boldsymbol{x}\geq 0} \inf_{\boldsymbol{\zeta}\in\mathcal{U}} \sum_{i=1}^{n_y} (p_i - c_i)x_i + \min(-b_i(\zeta_i - x_i), (s_i - p_i)(x_i - \zeta_i))$$

- Correlated demand : $\zeta \in \left\{ \zeta \,\middle|\, \begin{array}{c} \exists\delta,\ \|\delta\|_\infty \leq 1,\ \|\delta\|_1 \leq \Gamma \\ \zeta = \bar{\zeta} + Q\delta \end{array} \right\}$

Newsvendor Problem Correlated Demand			Gamma 30%	50%	70%	100%
10 items	Worst-case Profit (RO)	Avg Gap (%) - AARC	1,30%	1,62%	0,62%	**0,00%**
		Avg CPU time (s) - AARC	1,36	1,37	1,39	1,40
		Avg CPU time (s) - C&CG	115,87	145,90	178,41	126,36
	Worst-case Absolute Regret (WCARM)	Avg Gap (%) - AARC	3,16%	0,87%	0,16%	**0,00%**
		Avg CPU time (s) - AARC	1,40	1,41	1,43	1,44
		Avg CPU time (s) - C&CG	135,88	177,48	164,58	120,68
	Worst-case Relative Regret (WCRRM)	Avg Absolute Gap (%) - AARC	0,30%	0,13%	0,09%	**0,00%**
		Avg CPU time (s) - AARC	0,29	0,31	0,34	0,36
		Avg CPU time (s) - C&CG	208,30	262,04	258,86	157,77
20 items	Worst-case Profit (RO)	Avg Gap (%) - AARC	0,62%	0,52%	0,10%	**0,00%**
		Avg CPU time (s) - AARC	1,66	1,72	1,80	1,85
		Avg CPU time (s) - C&CG	286,28	451,72	582,15	314,32
	Worst-case Absolute Regret (WCARM)	Avg Gap (%) - AARC	0,66%	0,05%	0,01%	**0,00%**
		Avg CPU time (s) - AARC	1,94	1,96	2,00	2,06
		Avg CPU time (s) - C&CG	428,34	576,56	500,78	248,50
	Worst-case Relative Regret (WCRRM)	Avg Absolute Gap (%) - AARC	0,07%	0,05%	0,02%	**0,00%**
		Avg CPU time (s) - AARC	0,77	0,86	0,95	1,01
		Avg CPU time (s) - C&CG	717,13	2 681,47	6 287,90	567,82

Avg denotes the average of 10 randomly generated instances

Production-Transportation problem

(Bertsimas et al. [2010])

$$\underset{0\leq \boldsymbol{x}\leq 1}{\text{minimize}} \ \max_{\boldsymbol{\zeta}\in\mathcal{U}} h(\boldsymbol{x},\boldsymbol{\zeta}) \qquad \text{where} \qquad h(\boldsymbol{x},\boldsymbol{\zeta}) := \min_{\boldsymbol{y}\geq 0} \ \boldsymbol{c}^T\boldsymbol{x} + \sum_{ij}\zeta_{ij}y_{ij}$$

$$\text{s.t.} \quad \sum_i y_{ij} = d_j \ , \ \forall j$$

$$\sum_j y_{ij} = x_i \ , \ \forall i$$

Production-Transportation Problem			Gamma 30%	50%	70%	100%
5 facilities 10 customers	Worst-case Cost (RO)	Avg CPU time (s) - Exact	0,98	1,17	1,47	1,79
	Worst-case Absolute Regret (WCARM)	Avg Gap (%) - AARC	6,71%	7,21%	5,68%	4,97%
		Avg CPU time (s) - AARC	19,69	19,59	19,38	20,17
		Avg CPU time (s) - C&CG	42,86	65,40	93,22	95,80
	Worst-case Relative Regret (WCRRM)	Avg Absolute Gap (%)	0,39%	0,66%	0,78%	0,79%
		Avg CPU time (s) - AARC	23,29	24,60	23,93	23,90
		Avg CPU time (s) - Exact	299,70	555,37	1 000,34	1 563,99
7 facilities 14 customers	Worst-case Cost (RO)	Avg CPU time (s) - Exact	3,02	3,90	4,96	5,98
	Worst-case Absolute Regret (WCARM)	Avg Gap (%) - AARC	4,14%	4,54%	4,59%	4,21%
		Avg CPU time (s) - AARC	442,89	373,17	318,67	296,42
		Avg CPU time (s) - C&CG	3 425,35	8 365,05	6 967,85	7 468,70
	Worst-case Relative Regret (WCRRM)	Avg Absolute Gap (%) - AARC	-	-	-	-
		Avg CPU time (s) - AARC	352,46	319,68	346,89	451,33
		Avg CPU time (s) - Exact	**>14,400**	**>14,400**	**>14,400**	**>14,400**

Avg denotes the average of 10 randomly generated instances

Outline

- Introduction
- Regret minimization is as difficult as Two-Stage Robust Optimization
- Optimality of Affine Decision Rules
- Numerical experiments
- **Conclusion**

Take-away messages

- In two stages setting, regret minimization and robust optimization are closely related modelling paradigms in terms of solution/ approximation schemes
  - Many TSLRO schemes can be applied in regret minimization
  - **Affine decision rules perform surprisingly well and can even be proven exact**
- Future work:
  - Extending constant approximation results for AARC to regret minimization (Bertsimas & Goyal [2012], El Housny & Goyal [2018])
  - Worst-case regret minimization for multi-stage and/or mixed integer problems
  - Worst-case regret minimization for DRO problems

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Thank you for your attention