Course Title:

Quantitative risk management using robust optimization

Description:

Celebrating more than 20 years of renewed and flourishing interest in the robust optimization (RO) paradigm, this course will introduce students to the means of hedging risks in large decision problems where distribution assumptions cannot be made. More specifically, the students will become familiar with the main tools that are used in the application of the robust optimization paradigm: convex theory (duality theory, saddle point theorem, Karush-Kuhn-Tucker conditions), data-driven uncertainty sets design, adjustable decision rules, tractable reformulation and decomposition algorithms for problems of infinite size. In addition, the course will cover a set of practical applications where the use of such tools is called-for. At the end of the course, the students should be able to:

- identify a situation presenting a robust optimization problem,
- represent this problem under a suitable mathematical model,
- exploit available historical observations to adequately characterize the uncertainty
- identify robust solutions using cutting edge software and algorithms

Note that the techniques discussed in this course should appeal to both engineers and business analysts given that robust optimization is used pervasively in disciplines such as chemical, civil, computer, and electrical engineering, medicine, physics, machine learning, and operations management (see respectively [1]-[8]), to name a few.

In presenting the material of the course, the instructor will assume that students have been exposed to concepts of linear algebra, mathematical programming, probabilistic modeling and Monte-Carlo simulation. All implementations are performed using the Python Programming Language. The material of the course is drawn from a set of lecture notes [9] supported by two books (see [10] and [11]).

Instructor:

Erick Delage completed is Ph.D. at Stanford University in 2009, is currently professor in the Department of Decision Sciences at HEC Montréal, and is chairholder of Canada Research Chair in Decision Making under Uncertainty. He is internationally recognized for his expertise and contributions to robust optimization. He serves on the editorial board of several landmark journals in the field of Operations Research.

Course Outline:

Session 1 : Why a Surge of Interest in Robust Optimization?

- Introduction to decision making under uncertainty
- Examples of managerial problems where RO can make a difference
- Introduction to the RSOME library in Python

Sessions 2-3 : Robust Counterparts of Linear Programs

- Linear programming duality (convex sets/functions, separating hyperplane theorem, Farkas lemma)
- Robust counterparts of linear constraints
- Ellipsoidal and budgeted uncertainty sets

Session 4 : Data-driven Uncertainty Set Design

- The price of robustness
- Calibration based on chance constraints
- Calibration based on coherent risk measures
- Application to portfolio optimization problem

Session 5 : Adjustable Robust Linear Programming

- Conditions where there is no value in delaying the decision
- NP hardness of optimization of delayed decisions
- Column and constraint generation solution methods
- Application to facility location-transportation problem

Session 6 : Value of Flexibility using Tractable Decision Rules

- Affine Adjustable Robust Counterparts
- Trade-off between precision and computational burden

Session 7 : Robust Nonlinear Programming

- Exploiting Fenchel duality to derive decomposable counterparts
- Application to advertisement campaign planning
- Application to robust CVaR minimization when probabilities are uncertain

Session 8 : Globalized Robust Counterparts

- Application to a production problem controlling sensitivity to future drifts in prices
- Application to signal reconstruction with controlled deterioration

Sessions 9-10 : Distributionally Robust Optimization

- Introduce Ellsberg's paradox to motivate ambiguity aversion
- Finite dimensional reformulation using semi-infinite conic duality
- Data-driven distribution sets

Session 11 : Pareto Robust Optimization

- Introduction to Pareto efficiency
- Robust solutions are not always non-dominated
- Implications in terms of performance evaluation
- Methods for identifying Pareto robust solutions

Session 12 : A Survey of Recent RO Applications

- Students present recent applications of RO

Bibliography

[1] F. P. Bernardo and P. M. Saraiva. Robust optimization framework for process parameter and tolerance design. AIChE Journal, 44(9):2007–2017, 1998.

[2] M. P. Bendsøe, A. Ben-Tal, and J. Zowe. Optimization methods for truss geometry and topology design. Structural optimization, 7(3):141–159, 1994.

[3] M. Mani, A. K. Singh, and M. Orshansky. Joint design-time and post-silicon minimization of parametric yield loss using adjustable robust optimization. In 2006 IEEE/ACM International Conference on Computer Aided Design, pages 19–26, 2006.

[4] R. Jiang, J. Wang, Y. Guan, Robust Unit Commitment with Wind Power and Pumped Storage Hydro, IEEE Transactions on Power Systems, vol. 27, no. 2, pp. 800-10, 2012.

[5] M. Chu, Y. Zinchenko, S. G Henderson, and M. B Sharpe. Robust optimization for intensity modulated radiation therapy treatment planning under uncertainty. Physics in Medicine and Biology, 50(23):5463–5477, 2005.

[6] D. Bertsimas, O. Nohadani, and K. Meng Teo. Robust optimization in electromagnetic scattering problems. Journal of Applied Physics, 101(7):074507, 2007.

[7] H. Xu, C. Caramanis, and S. Mannor, Robust regression and Lasso, IEEE Transaction on Information Theory, 56, pp. 3561–3574, 2010.

[8] A. Ben-Tal, B. Do Chung, S. Reffy Mandala, T. Yao, Robust Optimization for Emergency Logistics Planning: Risk Mitigation in Humanitarian Relief Supply Chains, Transportation Res. B, vol. 45, no. 8, pp. 1177–1189, 2011.

[9] E. Delage. Quantitative Risk Management Using Robust Optimization: Lecture Notes, https://tintin.hec.ca/pages/erick.delage/MATH80624_LectureNotes.pdf, December 2021.

[10] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski. Robust Optimization. Princeton Series in Applied Mathematics, 2009.

[11] D. Bertsimas and D. den Hertog, Robust and Adaptive Optimization. Dynamic Ideas LLC, 2022.