Dynamics in Research Joint Ventures and R&D Collaborations*

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Abstract

We investigate the short- and long-term effects of different types of R&D collaborations on firms, consumers, and the industry. To that end, we consider a differentiated-product market in which firms compete à la Bertrand and invest in process innovation in order to lower the production cost over time. Investments are stochastic and there can be cartelization or competition strategies among firms at the moment of making the decision on the amount to invest in R&D. Our results show that in equilibrium, the long-run welfare is larger under a research joint venture than under other environments. Discounted present value profits increase with the level of the spillover but there are asymmetries that depend on the firms' asymmetry on marginal costs.

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1 Introduction

Collaboration among firms is sometimes seen as a form of collusion. However, when it comes to basic research and development (R&D), collaboration has been welcome by regulators.[1] The common wisdom is that process innovation might benefit collaborating firms (through lower costs) and consumers (through lower prices in the market). This paper quantifies those effects by analyzing the trade-off between the future benefits of increasing the likelihood of process innovation success and the costs associated with such investment.

We test one interpretation of Schumpeter (1942)'s hypothesis: innovation increases with market concentration. The empirical literature evaluating this hypothesis has been inconclusive but some recent studies using dynamic frameworks point out the benefits of market concentration and collaborations at the R&D level. Goettler and Gordon (2011) found evidence for Schumpeter's hypothesis in the PC market and product innovation. Gugler and Siebert (1986) conclude that research joint ventures (RJV) -a form of collaboration- in the semiconductor industry were associated with increases of industry market share. However, it is unclear the extent to which different levels of collaboration contribute to promote innovation and the social costs associated with them in a general setting.

In their seminal paper, D'Aspremont and Jacquemin (1988) showed that cooperation in R&D leads to higher investment levels in R&D than does noncooperation, and consequently to lower production costs when the knowledge spillover between firms is sufficiently high. Our paper belongs to this literature and compares the outcomes of different environments of collaboration (different degrees of information spillover) in a stochastic and dynamic setting of strategic continuous investments to reduce marginal costs. For each of those environments we consider two market structures at the process innovation level: competition and cartelization. At the product level we keep the assumption of competition following the literature.

Although the effect of different forms of R&D cooperation is well understood in two-stage models, the same effect has not been completely characterized in more complex dynamic models with stochastic investment. Following D'Aspremont and Jacquemin, a first group of subsequent studies investigated whether their results still hold under different assumptions. For instance, Kamien et al. (1992) obtained that a cartelized research joint venture (RJV), that is, firms share a single laboratory, yields the best performance in terms of R&D investments, consumer surplus as well as producer surplus. Suzumura (1992) established that, for a certain general demand function, neither competitive nor cooperative R&D equilibria are socially efficient. Amir and Wooders (1998) obtained that noncooperation in R&D may result in higher profits than does cooperation in an asymmetric equilibrium. Amir et al. (2008) showed that the d'Aspremont and Jacquemin's results still hold under a convex cost function that includes a fixed cost component. Salant and Shaffer (1998) considered asymmetric

[1]See Grossman and Shapiro (1986) for a discussion on the antitrust issues associated with R&D collaborations. The U.S. put in place the National Cooperative Research and Production Act of 1993 to regulate collaborations among firms at the R&D level. It "[establishes] a procedure under which joint ventures and standards development organizations that notify the Department of Justice and Federal Trade Commission of their cooperative ventures and standards development activities are liable for actual, rather than treble, antitrust damages.", DOJ (1993). The FTC frequently cites competition as a mechanism that deters innovation (Gilbert (2006)).

R&D investments and demonstrated that, even when there is no spillover, RJV increases social welfare.

A second stream of the literature considered that spillovers are endogenous. One way of putting it is to state that, to benefit from the rival's R&D, a firm must acquire some absorptive capacity, which depends on the own firm's R&D and on its R&D approach strategy. The R&D approach, which can be firm-specific or broad R&D, is decided in a first stage before choosing the expenditures in R&D (second stage) and output (third stage). The rationale for including absorptive capacity is best told by Kamien and Zang (2000) who argued that there are ample empirical evidence showing that without absorptive capacity, the firm cannot really benefit much from any available knowledge spilled over by the rivals. As these authors put it, to win a lottery, one needs to buy a ticket! One of their results is that firms adopt purely specific R&D strategies when they compete to offset spillovers, and broad strategies when they cooperate to maximize knowledge flows. Several authors have explored the interaction between the degree of this absorptive capacity and the intensity of the spillovers.[2] Another group of papers distinguishes explicitly, in one way or another, between innovative and absorptive research.[3]

All these papers share two assumptions, namely: (i) all firms in the industry are active in R&D; and (ii) the investment decision in R&D is made once.[4] We relax these two assumptions and allow for a dynamic cost-reduction R&D process. Whereas the product R&D literature has been dynamic, which is the essence of a patent race,[5] the literature adopting a dynamic game framework in a process R&D (cost-reduction) setting is sparse. Tolwinski and Zaccour (1995) considered a price-setting duopoly producing differentiated goods, where the players can learn by doing and from each other to decrease the unit production, which depends on

[2]Leahy and Neary (2007) showed that R&D investments increase absorptive capacity but decrease the incentive to cooperate. Hammerschmidt (2009) distinguished between two kinds of R&D investments: production-cost-reducing R&D and absorptive-capacity-improving R&D. They find that when spillovers are high, firms invest more to improve their absorptive capacity. Martin (2002) considered input and output spillovers, or appropriabilty, and dealt with an uncertain innovation process. He found that social welfare is maximized when input spillovers are high and appropriability is low. Silipo and Weiss (2005) studied R&D cooperation with spillovers and uncertainty and distinguished between incremental and offsetting spillovers. If spillovers are offsetting, then competition is preferred to cooperation, and the reverse if spillovers are incremental.

[3]Frascatore (2006), for instance, distinguished "basic research," which increases the firm's absorptive capacity, from "applied research," which reduces the firm's costs; see also Jin and Troege (2006), Kanniainen and Stenbacka (2000) and Ben Youssef et al. (2013) In this literature, investments in R&D necessarily increase the absorptive ability of firms (see also Poyago-Theotoky (1999), Wiethaus (2005), Grünfeld (2003), Kaiser (2002), and Milliou (2009)).

[4]Ceccagnoli (2005) and Abdelaziz et al. (2008) are the only studies to consider a heterogenous industry made of firms that invest in R&D and others that do not. Ceccagnoli (2005) analyzed the impact of the knowledge spillover to non-innovating firms, on the incentives of innovating firms to continue their cost-reducing R&D effort. Abdelaziz et al. (2008) confirmed the impact of free riding, namely, the presence of non-innovating firms in an industry leads to lower individual investments in R&D, to a lower collective level of knowledge and to a higher product price. They conclude that surfers (non-investing firms) presence in an industry could enhance R&D investment level and welfare in some region of the parameter space.

[5]It is often referred to this literature as tournament R&D where the winner takes all, which is the case of a patent.

accumulated knowledge. Breton et al. (2004) compared Bertrand and Cournot equilibria for a differentiated duopoly engaging in the process of R&D competition, and derived the conditions under which Bertrand competition is more efficient than Cournot competition. Shravan (2005) retained a dynamic model where a laggard firm can learn from the leader and characterized equilibrium strategies in this context. Breton et al. (2006) proposed a two-player infinite-horizon discrete-time game where the players invest in R&D in order to develop a new technology to reduce production costs.[6]

Most likely, the closest paper to ours is Cellini and Lambertini (2009). They considered a dynamic version of D'Aspremont and Jacquemin (1988) where firms may either undertake independent ventures or form a cartel for cost-reducing R&D investments. At the steady state, they showed that private and social incentives towards R&D cooperation coincide for all admissible levels of the technological spillovers characterizing innovative activity. We provide technical details on the differences between our approach and that of previous two-stage models of R&D cooperation in the next section.

Our model differs from those in the literature in a number of ways. First, whereas most papers adopt Cournot competition in the product market, we opt for Bertrand competition and therefore focus on pricing and market share issues. Second, although our model involves two firms, our demand functions integrate explicitly an outside option, which is more realistic than assuming that consumers are bound to deal with only these two companies. Third, our model is fully dynamic and stochastic. Whereas the first feature (dynamic) has been retained in some contributions in the past, the literature has systematically assumed absence of shocks in the industry.[7] We believe that adopting a dynamic game approach has a number of advantages with respect to a static one. First, in practice, process improvements are incremental and result from continuous and long-term investments in R&D. Unless we have a technological breakthrough, cost reduction is the fruit of learning and continuous investments in developing human and material resources (proxied by a single construct, namely R&D). A static approach cannot properly account for this cumulative effort because it does not distinguish between flow variables, e.g., investment in R&D, and stock variables, e.g., stock of knowledge. Second, firms meet and compete repeatedly in the market place and a dynamic setting is needed to represent and understand the long-term behavior and equilibrium in the industry. Policy makers interested in devising incentives (e.g., subsidies) to boost R&D efforts are clearly interested by the long-term effect on welfare of investing tax-payer dollars.

In terms of results, we provide a ranking on the expected welfare over time for the different R&D environments, under competition and under cartelization. We find that the largest gains occur when switching from R&D cartelization to RJV cartelization. This is in line with the findings in Kamien et al. (1992). Discounted present value profits increase with the level of the spillover but there are asymmetries that depend on the firms' asymmetry on marginal costs. We are also able to show the impacts of each environment on the transition paths for the different outcomes of interest: consumer surplus, prices, outside good market share, and

[6]They showed that firms do not invest in R&D if the knowledge level is too low. For an intermediate knowledge region where there are two pure Nash equilibria: either no firm does R&D or both firms do R&D.

[7]The only exception is probably Breton et al. (2006), but their setting is very different from ours in many respects.

profits. Even if some environments lead to the same outcome in the long-run, the rate at which those outcomes are attained significantly differs from one environment to another.

We also analyze the effects of different levels of likelihood of success of investment and of the appreciation rate for marginal costs on outcomes. Our results show that even though gains in profits occur when comparing environments with better levels of information sharing and collaboration, these gains can vary a lot depending on the specific point of the state space.

The rest of the paper is organized as follows. In Section 2 we compare in more detail previous models of R&D cooperation. We present our dynamic model in Section 3 and our main results in Section 4. In Section 5 we extend our results to a wide variety of parameter combinations. We conclude in Section 6.

2 Standard Two-stage Models of R&D Cooperation

In this section we describe the main results in the literature and point out the differences between our model and those of previous approaches as well as the consequences for our results.

The model in D'Aspremont and Jacquemin (1988) can be described as follows:

1. A two-stage game where the firms decide on their R&D expenditures in the first stage and compete à la Cournot (i.e., choose their output levels) in the second stage.

2. R&D efforts are process-oriented, that is, they are aimed at reducing the production cost of the homogenous product.

3. Each firm leaks part of its knowledge to competitors (the spillover effect) and, similarly, benefits gratuitously from its competitors' R&D efforts.

4. Firms are symmetric and active in R&D.

5. The model is deterministic.

This framework was extended by Kamien et al. (1992) by distinguishing between different types of R&D cooperation: coordination, information sharing, or both. On one hand, coordination is a type of cooperation at the level of the investment decision-making process. That is, the firms cooperate by making decisions that maximize the firms' joint profits. This is the set of cases we call cartelization. On the other hand, information sharing is cooperation regarding the use of new technologies used to lower the marginal cost of production conditional on the investment decision. More specifically, information sharing means that firms cooperate by sharing whatever findings they obtained from their investment.

Kamien et al. also extended that work by considering n firms instead of two, and a general concave R&D production function. They keep the assumption of Cournot competition for their main results.

In what follows, we explain the model using the notation in Kamien et al. (1992) but the underlying model is based on that of D'Aspremont and Jacquemin (1988). They consider a linear demand function and constant marginal costs that are the same for all firms. Each firm chooses a level of expenditure on R&D x_i that is aggregated in a common pool of investment $X_i = x_i + \gamma \sum_{j \neq i} x_j$ where $0 \leq \gamma \leq 1$ is the spillover parameter (this is the level of information sharing). The magnitude of unit cost reduction is given by $f(X_i)$ so that, after the R&D investment, firm i's new marginal cost is $c - f(X_i)$ where f is increasing in X_i and has standard properties to ensure the existence and uniqueness of the equilibrium.

We propose a model in which the R&D investment technology is stochastic. Specifically, our R&D investment technology reduces cost by one unit if there is success according to a probability distribution over the amount of investment. Kamien et al's R&D technology reduces cost in a deterministic manner by an amount equal to the level of investment.

When $\gamma = 1$ we are in a situation where the spillover is maximized, we identify this situation as an RJV. When $\gamma \in (0, 1)$ there is a partial spillover effect. In Kamien et al.'s setting, this parameter value has a partial and deterministic effect on the cost reduction for every firm. In our case, a successful reduction of cost by one unit for firm j has the probability γ of reducing firm i's cost by one unit.

In the second stage, all firms play Cournot competition. This yields optimal profits that depend on the parameter γ. As mentioned before, we do price-setting instead.

In the first stage, Kamien et al. consider four cases which correspond to those in rows 1 and 2 from Table 1. Column 1 represents the cases in which there is no coordination in the investment decision process. The cartelization column represents the cases of coordination (joint profits maximization). The degree of information sharing γ is represented in the rows of that table. Information sharing can be present or not in either of the competition and the cartelization environments

Table 1: Different levels of coordination and information sharing.

		market structure for coordination	
		competition	cartelization
level of info. sharing	no spillover	R&D $\gamma = 0$	
	partial spillover	$\gamma \in (0, 1)$	
	complete spillover	RJV $\gamma = 1$	

Given this 2-stage game setting, they found that the investments in each scenario can be ranked as follows:

$$\begin{aligned} X^{\text{RJV cart}} &\geq X^{\text{R\&D cart}} \geq X^{\text{RJV compet}}, \\ X^{\text{RJV cart}} &\geq X^{\text{R\&D compet}} \geq X^{\text{RJV compet}}, \end{aligned}$$

and

$$X^{\text{R\&D cart}} \geq X^{\text{R\&D compet}},$$

if and only if a certain restriction in the model parameters holds. For prices, they found a ranking of the different environments similar to those with investments except that all the inequalities are reversed. In terms of profits, they found that profits under RJV cartelization dominate the profits under each of the other three environments. The profits under R&D cartelization dominate those of R&D competition.

Therefore, there is in the literature a strong result about the dominance of RJV cartelization with respect to other environments. We want to assess the validity of these results in a full dynamic model of cooperation. How do these results, if any, change when firms compete in multiple periods and investment is no longer deterministic? We answer this question by finding optimal solutions to a multi-period dynamic model of cooperation with these characteristics: price competition, long-run solutions, different marginal costs, different levels of spillover effects, existence of an outside good, and stochastic investment.

3 A Dynamic Model of R&D and RJV

3.1 Competition

In this section, we consider the model of R&D competition in which each firm owns an R&D laboratory in order to innovate and lower the cost of production. We allow for information spillovers, that is, innovation by one firm might be leaked out to the other firm. When the information spillover is perfect (i.e., the information is always shared), then we are in the case of a research joint venture (RJV). Our baseline setup follows McGuire and Pakes (1994). In particular, our investment and competition processes closely follow their formulations. They provide an algorithm for computing Markov perfect Nash equilibria for a class of dynamic games that has been widely used in the literature.[8] One of our contributions in this paper is to extend and simulate this class of models in order to compare different R&D and RJV environments by specifying a flexible stochastic innovation process.

Demand. We start with the logit demand function of McFadden (1974).[9] This function can be derived from microeconomic principles as follows. The consumer i decides between purchasing one unit of good $j \in \{A, B\}$ or not buying any good at all in which case we say she opted for the *outside good*. She solves the problem

$$\max_{j,w} U(x_j, w) \quad \text{s.t.} \quad p_j + p_0 w = y$$

where U is her utility function, x_j are characteristics of the good j (which is why they only enter in the utility function but not in the budget constraint), p_j is the price, w is the amount of the outside good, and p_0 its price. Then, conditional on choosing one unit of good j, the

[8]For a list of some of the applications of this type of models see Section 7 in Doraszelski and Pakes (2007).

[9]This is the work that eventually led McFadden to win the Nobel Prize in Economics in 2000.

indirect utility functions are

$$U_j^*(x_j, p_j, p_0, y) = U\left(x_j, \frac{y - p_j}{p_0}\right)$$

and

$$U_0^*(p_0, y) = U\left(0, \frac{y - p_j}{p_0}\right)$$

if she chooses the outside option. Her final problem is discrete:

$$\max_{j \in \{A,B,0\}} \{U_j^*(x_j, p_j, p_0, y) = V_j(x_j, p_j, p_0, y) + \epsilon_j\}$$

where we have decomposed her utility function into an observable part V_j and an unobservable term (to the econometrician but known to the consumer) ϵ_j. We define the demand for good j for this consumer as

$$\begin{aligned} s_j(p_j, p_k) &= \Pr(U_j^* > U_k^* \quad \text{for } k \neq j) \\ &= \Pr(\epsilon_j - \epsilon_k > V_j - V_k \quad \text{for } k \neq j). \end{aligned}$$

If ϵ_j and ϵ_k are identically and independently distributed according to a type-I extreme-value distribution, then the random variable $\epsilon_j - \epsilon_k$ follows a logistic distribution and we obtain that

$$s_j(p_j, p_k) = \frac{e^{V_j}}{e^{V_0} + e^{V_j} + e^{V_k}}.$$

In particular, if we normalize $V_0 \equiv 0$ and parameterize the observable part of the utility function as $V_j = \theta - \lambda p_j$, we obtain

$$D_j(p_j, p_k) = m \frac{e^{\theta - \lambda p_j}}{1 + e^{\theta - \lambda p_j} + e^{\theta - \lambda p_k}}, \tag{1}$$

where $\{m, \theta, \lambda\}$ are market parameters. Specifically, $m > 0$ measures the size of the market and $\{\theta, \lambda\}$ reflect consumers' preferences. This functional form implies that $D_j(p_j, p_k) \in [0, m]$ and it implies as well the existence of an outside good market share given by $s_0 = 1/(1 + e^{\theta - \lambda p_j} + e^{\theta - \lambda p_k})$.

The effects of the presence of the outside good option in our model are of high relevance. First, we note that the parameter θ can be interpreted as the quality of the goods since this parameter can be seen as an aggregate of all the characteristics of the good other than the price. It can also be related to consumers' characteristics such as income but throughout the paper we will concentrate on its interpretation as the quality of the good. We also assume that this term is constant across goods.[10] It is clear that if θ increases, the market shares of

[10] Allowing for different qualities is not a major complication in this model. However, given the main purpose of our paper, we want to concentrate on the effects of investment on marginal costs. These effects are better understood if we keep quality constant across the goods.

goods A and B increase but that of the outside good decreases. This is intuitively correct since a higher quality product should be attractive to consumers who were not buying goods A and B before.

The introduction of the outside good is also useful for the interpretation of price increases. For, if both prices increase, fewer customers will opt to buy any of the two goods thus increasing the size of the outside good market share. This can be seen by looking at the expression for s_0. As we explain in the supply part of the model, we are interested in investigating the effects of different market environments on prices. If one of these environments implies large price increases, we expect a large number of customers to opt out and stop buying the goods. Those customers will be counted in the outside good market share and thus this allows for a complete characterization of the way the market is being segmented.[11]

The parameter λ captures the average consumers' sensitivity to changes in prices. Specifically, the own- and cross-price elasticities in our model are $-\lambda s_j(1-s_j)$ and $\lambda s_i s_j$, respectively. It is important to note that these elasticities are not linear in λ since the market shares are functions of λ themselves. However, it is clear that for given market shares values (from data for instance), it is the parameter λ what determines the size of all the elasticities in the demand system.

Profits. Using (1), at any period, firm j's profits are

$$\pi_j\left(p_j, p_k; c_j, c_k\right) = D_j\left(p_j, p_k\right)\left(p_j - c_j\right),$$

where $c_j \in \{0, 1, 2, ..., M\}$ is firm j's marginal cost of production. As it will become clear when we present the expressions for the value functions for the dynamic game, at any given time period t the only decision variable is the amount of investment, which is then realized conditional on the contemporaneous levels of marginal costs (the state variables). Depending on the success of the investments and the industry shock, marginal costs levels may change in the next period but not in the current period. Thus, the contemporaneous profits at each point of the space of marginal costs combinations do not depend on the level of investments at $t-1$. This allows us to simply compute the matrix of static profits beforehand, store these values, and use this matrix accordingly when we update the value functions. This is one of the characteristics in the McGuire and Pakes (1994) model.

Let $\Pi\left(c_j, c_k\right)$ be firm j's instantaneous profit corresponding to the static Bertrand game.[12] Note that Π is not indexed by j. Hence, for any combination of marginal costs (c_A, c_B), $\Pi\left(c_A, c_B\right)$ is the profit for firm A and $\Pi\left(c_B, c_A\right)$ is the profit for firm B. That is, the matrix with the values for the static profits for a given firm is the transpose of the corresponding matrix of the other firm.

Investment. Each period, firm j purchases x_j units of investment in order to yield process innovation. Specifically, $\tilde{\imath}_j$ is a random variable with support $\{-1, 0\}$ such that,

[11] In addition, the use of the outside good market option is important in empirical applications as it allows for the estimation of θ and λ by dividing each inside good market share expression by the outside good market share (see Berry (1994)).

[12] That is, for $j \in \{A, B\}, j \neq k$, $\Pi\left(c_j, c_k\right) = D_j\left(p_j^*, p_k^*\right)\left(p_j^* - c_j\right)$ where the pair $\{p_A^*, p_B^*\}$ is the Bertrand equilibrium defined as $p_j^* = \arg\max_{p_j>0} D_j\left(p_j, p_k^*\right)\left(p_j - c_j\right)$. For all $\{c_A, c_B\}$, there exists a unique Bertrand-Nash equilibrium (Caplin and Nalebuff (1991)).

conditional on the level of investment $x_j \geq 0$,

$$\phi(x_j) \equiv \Pr[\tilde{\imath}_j = -1 | x_j] = \frac{\alpha x_j}{1 + \alpha x_j},$$

is the probability of firm j achieving process innovation (a decrease in marginal costs). Here, $\alpha > 0$ is a parameter that reflects the effectiveness of investment to generate process innovation so that αx_j is firm j's level of effective investment. From (3.1), a higher level of effective investment increases firm j's probability to achieve process innovation.

Innovation Process and Cost Reduction. We now describe how process innovation translates into cost reduction. There are two sources of randomness in the innovation process. The first source is whether the innovation process is successful or not. Formally, let $\tilde{f}(\tilde{\imath}_j, \tilde{\imath}_k)$ capture the effect of present innovations on firm j's cost reduction in the next period, where $\imath_{\cdot}$ is a binary random variable. The tilde sign on $(\tilde{\imath}_j, \tilde{\imath}_k)$ indicates realizations of the random variables. The tilde sign for the function itself captures the uncertainty about information sharing. We now explain in detail the functional forms we adopt to consider all cases of interest.

Let us first focus on cases in which f is deterministic. First, if there is no information sharing, then firm j reduces cost when its own laboratory innovates, i.e., $f(\tilde{\imath}_j, \tilde{\imath}_k) = \tilde{\imath}_j \in \{-1, 0\}$, (in this case f is simply a projection of the two random variables onto the one that corresponds to this particular firm). Second, if there is information sharing with duplicate innovations, then firm j reduces cost when either laboratory innovates, but does not gain from learning about firm k's innovation when both firms innovate, i.e., $f(\tilde{\imath}_j, \tilde{\imath}_k) = \min\{\tilde{\imath}_j, \tilde{\imath}_k\} \in \{-1, 0\}$.

The second source of randomness is an industry cost shock $\tilde{\eta}$ with support $\{0, 1\}$ and it is assumed to have the following exogenous distribution

$$\Pr[\tilde{\eta} = 1] = \delta \in [0, 1],$$

where δ can be interpreted as the probability of cost appreciation. Firms cannot control this random variable even if their levels of investment are high. There is no correlation either between this shock and the success of investment random variable, i.e. we assume that the random variables $(\tilde{\imath}_1, \tilde{\imath}_2, \tilde{\eta})$ are independent from each other.

Conditional on $f(\tilde{\imath}_j, \tilde{\imath}_k)$, we can now define the law of motion for cost. Letting c_j and c'_j be firm j's marginal cost this period and next period respectively, the stochastic evolution of firm j's cost depends on both firm-specific and industry shocks. Moreover, the firm-specific shock depends on the level of investment. Given firm j's present marginal cost and given $f(\cdot)$, the marginal cost in the next period evolves stochastically as

$$\tilde{c}'_j | c_j = \min\{\max\{c_j + f(\tilde{\imath}_j, \tilde{\imath}_j) + \tilde{\eta}, 0\}, M\}, \tag{2}$$

where $\tilde{\imath}_j$ is the firm-specific shock, $\tilde{\eta}$ is the industry shock, and M is the exogenous maximum level of marginal cost. Recall that a tilde sign distinguishes a random variable from its realization. A prime sign indicates a variable in the subsequent period.

In equation (2) we assume that the evolution of a firm's marginal cost depends on both endogenous and exogenous shocks. Indeed, the exogenous shock $\tilde{\eta}$ encompasses outside factors (e.g., changes in the prices of inputs) affecting the industry. However, the endogenous shocks $\tilde{\imath}_j$ are linked to the firms' investment decisions and are thus firm-specific. Finally, in equation (2), given the distribution of the shocks, the max and min operators ensure that the marginal cost remains on the support of integers between 0 and M.

Having defined how the innovation process translates into cost reduction for a given functional form for f, we extend the model to allow for randomness about the presence of information sharing. Formally, in order to account for the two possible functional forms of $f(\tilde{\imath}_j, \tilde{\imath}_k) \in \{\tilde{\imath}_j, \min\{\tilde{\imath}_j, \tilde{\imath}_k\}\}$, we now specify a distribution over the presence of information sharing. Specifically, with probability $\gamma \in [0, 1]$, information about innovation process is leaked out. On the one hand, setting $\gamma = 0$ is the case of no information sharing, which in the literature is referred to as the R&D case. On the other hand, setting $\gamma = 1$ yields information sharing, often referred in the literature as a Research Joint Venture (RJV). Hence, using (2), the marginal cost in the next period evolves stochastically as

$$\tilde{c}'_j | c_j = \min\{\max\{c_j + \tilde{f}(\tilde{\imath}_j, \tilde{\imath}_k) + \tilde{\eta}, 0\}, M\},$$

where conditional on $(\tilde{\imath}_j, \tilde{\imath}_k)$, $\tilde{f}(\tilde{\imath}_j, \tilde{\imath}_k)$ is the random function with support $\{\tilde{\imath}_j, \min\{\tilde{\imath}_j, \tilde{\imath}_k\}\}$ and corresponding probabilities $\{1 - \gamma, \gamma\}$. That is, conditional on innovation outcomes $(\tilde{\imath}_j, \tilde{\imath}_k)$, information is leaked out with probability γ. This is an ex ante probability: we fix this probability at time 0 and its value does not change despite changes in other components of the market environment. Each of the different values for γ correspond to each of the different scenarios depicted in Table 1 in Section 2.

Value Function. For $\{j, k\} \in \{A, B\}$, given x_k, firm j's value function for an infinite-period horizon is

$$v\left(c_j, c_k\right) = \max_{x_j \geq 0} \left\{\Pi\left(c_j, c_k\right) - dx_j + \beta \mathbf{E}[v^{\tau-1}(\tilde{c}'_j, \tilde{c}'_k) | c_j, c_k, x_j, x_k]\right\}, \tag{3}$$

where $d > 0$ is the cost per unit of investment, $\beta \in (0, 1)$ is the discount factor and $E[v^{\tau-1}(\tilde{c}'_j, \tilde{c}'_k) | c_j, c_k, x_j, x_k]$ is the expected continuation value function. Note that we could also write equation (3) by taking the term $\Pi\left(c_j, c_k\right)$ outside the max operator, this makes explicit the fact that the static profits can be calculated only once because they do not depend on x_j or x_k.

From (3), the two firms interact strategically through the continuation value function. Before proceeding with a definition of the equilibrium, we describe in details the expected continuation value function. Given that c_j is bounded between 0 and M, more notation is introduced to account for changes in the marginal cost at the boundaries. Indeed, if $c_j = 0$, then $c'_j = 0$ for $(i_j, i_k, \eta) = (-1, i_k, 0), i_k \in \{-1, 0\}$ since more process innovation in the absence of cost appreciation does not lead to lower cost when cost is already zero. Similarly, if $c_j = M$, then $c'_j = M$ for $(i_j, i_k, \eta) = (0, 0, 1)$ since a positive industry shock in the absence of a negative firm-specific shock does not lead to a higher cost when cost is already at the maximum value. Formally, let

$$c_j^+ \equiv \min\{c_j + 1, M\}, \tag{4}$$

$$c_k^+ \equiv \min\{c_k + 1, M\}, \tag{5}$$

$$c_j^- \equiv \max\{c_j - 1, 0\}, \text{ and} \tag{6}$$

$$c_k^- \equiv \max\{c_k - 1, 0\}. \tag{7}$$

These account for all possible changes in marginal cost given (c_j, c_k).

Using (4), (5), (6), and (7), we describe the support of future payoffs with their corresponding probabilities. Specifically, at each period, given levels of investment $\{x_A, x_B\}$, the support of $\{\tilde{\imath}_1, \tilde{\imath}_2, \tilde{\eta}\}$ has eight elements. Indeed, each firm may succeed in achieving process innovation, i.e., $(i_1, i_2) \in \{(-1,-1), (-1,0), (0,-1), (0,0)\}$, and for each of the four outcomes about firms' success, there are two outcomes for the industry-wide appreciation shock, i.e., $\eta \in \{0, 1\}$. Hence, with probability $\phi(x_j)\phi(x_k)$, both firms achieve process innovation, i.e., $(i_1, i_2) = (-1, -1)$, which yields expected future payoffs

$$\begin{aligned} \Delta_{j,1} \;&\equiv\; (1-\gamma)\left[\delta v\left(c_j, c_k\right) + (1-\delta) v(c_j^-, c_k^-)\right] \\ &\quad\; +\gamma[\delta v\left(c_j, c_k\right) + (1-\delta) v(c_j^-, c_k^-)] \\ &\equiv\; \delta v\left(c_j, c_k\right) + (1-\delta) v(c_j^-, c_k^-), \end{aligned} \tag{8}$$

which takes into account the probability $\delta \in [0, 1]$ of an industry-wide appreciation cost affecting both firms as well as the probability of information sharing. That is, with probability $\phi(x_j)\phi(x_k)\left(1-\gamma\right)\delta$, both firms innovate without information sharing, but are hit with a cost appreciation shock, which yields no changes in the state variable, i.e., $\left(c_j', c_k'\right) = (c_j, c_k)$ with the corresponding expected stream of payoffs $v\left(c_j, c_k\right)$. With probability $\phi(x_j)\phi(x_k)\left(1-\gamma\right)(1-\delta)$, both firms innovate without information sharing and there is no industry-wide cost appreciation, i.e., $\left(c_j', c_k'\right) = \left(c_j^-, c_k^-\right)$ with the corresponding expected stream of payoffs $v\left(c_j^-, c_k^-\right)$. Moreover, with probability $\phi(x_j)\phi(x_k)\gamma\delta$, both firms obtain duplicate innovation with information sharing, but are hit with a cost appreciation shock, which yields no changes in the state variable, i.e., $\left(c_j', c_k'\right) = (c_j, c_k)$ with the corresponding expected stream of payoffs $v\left(c_j, c_k\right)$. Finally, with probability $\phi(x_j)\phi(x_k)\gamma(1-\delta)$, both firms obtain duplicate innovation with information sharing and there is no industry-wide cost appreciation, i.e., $\left(c_j', c_k'\right) = (c_j^-, c_k^-)$ with the corresponding expected stream of payoffs $v(c_j^-, c_k^-)$.

The same logic applies to the remaining three cases of firms' innovation. That is, with probability $\phi(x_j)(1-\phi(x_k))$, only firm j innovates, i.e., $(i_1, i_2) = (-1, 0)$ which yields expected future payoffs

$$\begin{aligned} \Delta_{j,2} \;&\equiv\; (1-\gamma)[\delta v(c_j, c_k^+) + (1-\delta) v(c_j^-, c_k)] \\ &\quad\; +\gamma[\delta v\left(c_j, c_k\right) + (1-\delta) v(c_j^-, c_k^-)]. \end{aligned} \tag{9}$$

With probability $(1-\phi(x_j))\phi(x_k)$, only firm k innovates, i.e., $(i_1, i_2) = (0, 1)$ which yields expected future payoffs

$$\begin{aligned} \Delta_{j,3} \;&\equiv\; (1-\gamma)[\delta v(c_j^+, c_k) + (1-\delta) v(c_j, c_k^-)] \\ &\quad\; +\gamma[\delta v\left(c_j, c_k\right) + (1-\delta) v(c_j^-, c_k^-)]. \end{aligned} \tag{10}$$

Finally, with probability $(1-\phi(x_j))(1-\phi(x_k))$, neither firm is successful in their R&D, which yields expected future payoffs

$$\begin{aligned}\Delta_{j,4} &\equiv (1-\gamma)[\delta v(c_j^+, c_k^+) + (1-\delta)v\,(c_j, c_k)] \\ &\quad +\gamma[\delta v(c_j^+, c_k^+) + (1-\delta)v\,(c_j, c_k)] \\ &\equiv \delta v(c_j^+, c_k^+) + (1-\delta)v\,(c_j, c_k)\,, \end{aligned} \tag{11}$$

where with probability δ, the lack of process innovation along with a cost appreciation leads to higher cost with expected future profits of $v(c_j^+, c_k^+)$.

In summary, each $\Delta_{j,l}, l = 1, \ldots, 4$ is the expected value of $v(\cdot, \cdot)$ over the distribution of the industry shock and of the spillover conditional on a specific combination of success of investment from both firms.

Using (8), (9), (10), and (11) with corresponding probabilities, the expected continuation value function in (3) is thus defined by

$$\begin{aligned}\mathbf{E}[v(\tilde{c}_j', \tilde{c}_k')|c_j, c_k, x_j, x_k] &= \phi(x_j)\phi(x_k) \cdot \Delta_{j,1} \\ &\quad +\phi(x_j)(1-\phi(x_k)) \cdot \Delta_{j,2} \\ &\quad +(1-\phi(x_j))\phi(x_k) \cdot \Delta_{j,3} \\ &\quad +(1-\phi(x_j))(1-\phi(x_k)) \cdot \Delta_{j,4}. \end{aligned} \tag{12}$$

Given an initial state (c_j, c_k), expression (12) summarizes all possible changes in the states corresponding to investment levels (x_j, x_k).

3.1.1 Equilibrium and Numerical Approach

Equilibrium. Next, we define the Markov-perfect Nash equilibrium (MPNE). Let $X(c_j, c_k)$ be firm j's investment-strategy of the MPNE. We focus on symmetric investment-strategy functions. In other words, for any (c_A, c_B), $X(c_A, c_B)$ is firm A's level of investment and thus $X(c_B, c_A)$ is firm B's level of investment.

Definition The tuple $\{X(c_A, c_B), X(c_B, c_A)\}$ is a MPNE for a game of infinite-period horizon if, for $j, k \in \{A, B\}$ and given $X(c_k, c_j)$,

$$X(c_j, c_{3-j}) = \arg\max_{x_j \geq 0} \left\{\Pi\left(c_j, c_{3-j}\right) - dx_j + E[V(,)|\cdot]\right\},$$

where for any $y, z \in \{1, 2, ..., M\}$,

$$\begin{aligned}E[V(,)|\cdot] = {} & \beta\phi(X(y,z))\phi(X(z,y)) \cdot \big(\delta V\,(y,z) + (1-\delta)V(y^-, z^-)\big) \\ & + \beta\phi\left(X(y,z)\right)\left(1-\phi\left(X(z,y)\right)\right) \cdot \big[(1-\gamma)(\delta V(y,z^+) + (1-\delta)V(y^-, z)) \\ & +\gamma(\delta V\,(y,z) + (1-\delta)V(y^-, z^-))\big] \\ & + \beta(1-\phi\left(X(y,z)\right))\phi\left(X(z,y)\right) \cdot \big[(1-\gamma)(\delta V(y^+, z) + (1-\delta)V(y, z^-)) \\ & +\gamma\left(\delta V\,(y,z) + (1-\delta)V(y^-, z^-)\right)\big] \\ & + \beta(1-\phi\left(X(y,z)\right))(1-\phi\left(X(z,y)\right)) \cdot \big(\delta V(y^+, z^+) + (1-\delta)V\,(y,z)\big)\,.\end{aligned}$$

The following proposition provides the reaction function necessary to characterize the equilibrium.

Proposition 3.1. *For $j, k \in \{A, B\}$, given x_k, firm j's reaction function is*

$$R(x_k) = \begin{cases} 0, & \text{if } G < 0 \\ \max\left\{-\frac{1}{\alpha} + \sqrt{\frac{\beta}{\alpha d}}\sqrt{G}, 0\right\}, & \text{if } G \geq 0, \end{cases} \tag{13}$$

where $G = \phi(x_k)(\Delta_{j,1} - \Delta_{j,3}) + (1 - \phi(x_k))(\Delta_{j,2} - \Delta_{j,4})$.

Proof See Appendix.

The interpretation of the two inequalities that define the reaction function is as follows. $\Delta_{j,1} - \Delta_{j,3}$ is the net gains when both firms innovate relative to the situation in which only firm k innovates. $\Delta_{j,2} - \Delta_{j,4}$ is the net gains when only firm j innovates relative to the outcome when none of the firm innovates. Therefore, G is the average of those two net gains weighted by the distribution of the probability of firm k's success. This interpretation becomes useful when determining whether the slope of the reaction function $R(x_k)$ is positive or negative, which determines whether the investments from each firm are strategic complements or strategic substitutes, respectively. To see this, we compute the derivative of $R(x_k)$ and obtain that $\text{sgn}(R(x_k)') = \text{sgn}(\Delta_{j,1} - \Delta_{j,3} - (\Delta_{j,2} - \Delta_{j,4}))$. Then, we have strategic complements when $\Delta_{j,1} - \Delta_{j,3} > \Delta_{j,2} - \Delta_{j,4}$ and strategic substitutes when the inequality is reversed. Note that the strategic complement investments arise when the relative gains from both firms innovating are greater than the relative gains when only firm j innovates. Conversely, strategic substitute investments arise when the relative gains from both firms innovating are weaker than when only firm j innovates, in which case this firm is better off lowering its investment when the other firm invests more.[13]

Numerical Approach. Since analytical solutions do not exist for these types of models, we make use of the techniques in Ericson and Pakes (1995) and McGuire and Pakes (1994) (PM).[14] We also compute solutions based on an algorithm proposed first by Levhari and Mirman (1980) (LM) for these types of problems. We obtain the exact same solutions under both approaches.

We present the algorithm for the LM approach but very similar notation would lead to the exposition of the PM algorithm. LM consists of computing the equilibrium for any finite horizon and increasing the horizon (making use of the computation for shorter horizons) until convergence is attained. Formally, for $\tau = 0$, for all (c_j, c_k), $X^0(c_j, c_k) = 0$ and

$$V^0(c_j, c_k) = \Pi(c_j, c_k).$$

[13]This interpretation is consistent with our numerical results. In the vast majority of cases we find evidence for strategic complements at the converged reaction curves. In the R&D competition environment we find evidence of strategic substitutes.

[14]See Doraszelski and Pakes (2007) for further details.

For $\tau > 1$, given $V^{\tau-1}(c_j, c_k)$, using (13), when the second-order condition is satisfied, i.e.,

$$\frac{\alpha X^{\tau-1}(c_k, c_j)\left(\Delta_{j,1}^{\tau-1} - \Delta_{j,3}^{\tau-1}\right)}{1 + \alpha X^{\tau-1}(c_k, c_j)} + \frac{\Delta_{j,2}^{\tau-1} - \Delta_{j,4}^{\tau-1}}{1 + \alpha X^{\tau-1}(c_k, c_j)} > 0,$$

the investment strategy $\{X_A^\tau, X_B^\tau\} \equiv \{X^\tau(c_j,c_k), X^\tau(c_k,c_j)\}$ is defined by

$$X_A^\tau = \max\left\{-\frac{1}{\alpha} + \sqrt{\frac{\beta}{\alpha d}}\sqrt{\frac{\alpha X_B^\tau\left(\Delta_{A,1}^{\tau-1} - \Delta_{A,3}^{\tau-1}\right)}{1 + \alpha X_B^\tau} + \frac{\Delta_{A,2}^{\tau-1} - \Delta_{A,4}^{\tau-1}}{1 + \alpha X_B^\tau}}, 0\right\},$$

and

$$X_B^\tau = \max\left\{-\frac{1}{\alpha} + \sqrt{\frac{\beta}{\alpha d}}\sqrt{\frac{\alpha X_A^\tau\left(\Delta_{B,1}^{\tau-1} - \Delta_{B,3}^{\tau-1}\right)}{1 + \alpha X_A^\tau} + \frac{\Delta_{B,2}^{\tau-1} - \Delta_{B,4}^{\tau-1}}{1 + \alpha X_A^\tau}}, 0\right\},$$

where, using (8), (9), (10), and (11), for $j \in \{A, B\}$,

$$\begin{aligned}\Delta_{j,1}^{\tau-1} \equiv\ & (1-\gamma)\left[\delta V^{\tau-1}(c_j, c_k) + (1-\delta)V^{\tau-1}(c_j^-, c_k^-)\right] \\ & +\gamma[\delta V^{\tau-1}(c_j, c_k) + (1-\delta)V^{\tau-1}(c_j^-, c_k^-)],\end{aligned}$$

$$\begin{aligned}\Delta_{j,2}^{\tau-1} \equiv\ & (1-\gamma)[\delta V^{\tau-1}(c_j, c_k^+) + (1-\delta)V^{\tau-1}(c_j^-, c_k)] \\ & +\gamma[\delta V^{\tau-1}(c_j, c_k) + (1-\delta)V^{\tau-1}(c_j^-, c_k^-)],\end{aligned}$$

$$\begin{aligned}\Delta_{j,3}^{\tau-1} \equiv\ & (1-\gamma)[\delta V^{\tau-1}(c_j^+, c_k) + (1-\delta)V^{\tau-1}(c_j, c_k^-)] \\ & +\gamma[\delta V^{\tau-1}(c_j, c_k) + (1-\delta)V^{\tau-1}(c_j^-, c_k^-)],\end{aligned}$$

and

$$\Delta_{j,4}^{\tau-1} \equiv \delta V^{\tau-1}(c_j^+, c_k^+) + (1-\delta)V^{\tau-1}(c_j, c_k)$$

depend on the value function computed at the $(\tau - 1)$ iteration. Finally,

$$\begin{aligned}V^\tau(c_j, c_k) = \ & \Pi(c_j, c_k) - dX^\tau(c_j,c_k) \\ & +\beta\phi(X^\tau(c_j,c_k))\phi(X^\tau(c_k,c_j)) \cdot \Delta_{j,1}^{\tau-1} \\ & +\beta\phi(X^\tau(c_j,c_k))(1 - \phi(X^\tau(c_k,c_j))) \cdot \Delta_{j,2}^{\tau-1} \\ & +\beta(1 - \phi(X^\tau(c_j,c_k)))\phi(X^\tau(c_k,c_j)) \cdot \Delta_{j,3}^{\tau-1} \\ & +\beta(1 - \phi(X^\tau(c_j,c_k)))(1 - \phi(X^\tau(c_k,c_j))) \cdot \Delta_{j,4}^{\tau-1}.\end{aligned}$$

The iteration continues until convergence in $X(c_j, c_k)$ and $V(c_j, c_k)$ is reached for all (c_j, c_k).

3.2 Cartelization

Having studied the effect of information sharing on the dynamics of the industry, we extend the analysis to the case of cartelization. Here, the two firms coordinate their R&D activities, which internalizes the investment externality. Note that the pricing game and the stochastic process for cost innovation are the same as in the case of R&D competition. The only difference is that there is no longer a game in investment decisions. That is, the cartel's value function for an infinite-period horizon is

$$w(c_A, c_B) = \max_{x_A, x_B \geq 0} \left\{ \Pi(c_A, c_B) + \Pi(c_B, c_A) - d(x_A + x_B) + \beta \mathbf{E}[w(\tilde{c}_1', c_B)|c_A, c_B, x_A, x_B] \right\}, \tag{14}$$

where

$$\begin{aligned} \mathbf{E}[w(c_A, c_B)|c_A, c_B, x_A, x_B] \quad = \quad & \phi(x_A)\phi(x_B) \cdot \Theta_1 + \phi(x_A)(1-\phi(x_B)) \cdot \Theta_2 \\ & +(1-\phi(x_A))\phi(x_B) \cdot \Theta_3 \\ & +(1-\phi(x_A))(1-\phi(x_B)) \cdot \Theta_4, \end{aligned} \tag{15}$$

$$\Theta_1 \equiv \delta w(c_A, c_B) + (1-\delta) w(c_A^-, c_B^-),$$

$$\begin{aligned} \Theta_2 \quad \equiv \quad & (1-\gamma)[\delta w(c_A, c_B^+) + (1-\delta) w(c_A^-, c_B)] \\ & +\gamma[\delta w(c_A, c_B) + (1-\delta) w(c_A^-, c_B^-)], \end{aligned}$$

$$\begin{aligned} \Theta_3 \quad \equiv \quad & (1-\gamma)[\delta w(c_A^+, c_B) + (1-\delta) w(c_A, c_B^-)] \\ & +\gamma[\delta w(c_A, c_B) + (1-\delta) w(c_A^-, c_B^-)], \end{aligned}$$

and

$$\Theta_4 \equiv \delta w(c_A^+, c_B^+) + (1-\delta) w(c_A, c_B).$$

Note that the only difference between (3) and (14) resides in the instantaneous profits. Under R&D competition, firm j's instantaneous profits are $\Pi(c_j, c_k) - dx_j$ whereas, under R&D cartelization, the firms' combined instantaneous profits are $\Pi(c_A, c_B) + \Pi(c_B, c_A) - d(x_A + x_B)$. Hence, the distribution of the continuation value function remains unchanged, only the payoffs change. To see this, compare (12) and (15). The probabilities are the same but the realized payoffs are different. Since at the optimum, $x_A = x_B$, the maximization problem is rewritten with one choice variable.

Similarly to the interpretation of $\Delta_{j,l}, l = 1, \ldots, 4$, each Θ_l is the expected value of the cartel's value function over the distribution of the industry-wide shock and of the spillover conditional on a particular combination of success of investment in each firm.

Definition In a cartel,[15]

$$X^C(c_A, c_B) = \arg\max_{x \geq 0} \left\{ \Pi(c_A, c_B) + \Pi(c_B, c_A) - 2dx + \beta \mathbf{E}[W(c_A, c_B)|c_A, c_B, x] \right\},$$

[15]Here, for cartelization, we need not to have $X(c_2, c_1)$ and $X(c_1, c_2)$. The distinction is only necessary for the competition cases as we need to distinguish between the own cost and the rival's cost.

where

$$\begin{aligned}\mathbf{E}[W(c_A,c_B)|c_A,c_B,x] &= \phi(x)\phi(x)\cdot\Theta_1\\ &+\phi(x)(1-\phi(x))\cdot\Theta_2\\ &+(1-\phi(x))\phi(x)\cdot\Theta_3\\ &+(1-\phi(x))(1-\phi(x))\cdot\Theta_4.\end{aligned}$$

The following proposition characterizes the solution to the cartelization problem.

Proposition 3.2. *$X^C(c_A,c_B)$ is defined by the third-degree polynomial*

$$Ax^3+Bx^2+Cx+D=0, \tag{16}$$

where

$$\begin{aligned}A &= -2d\alpha^3,\\ B &= -6d\alpha^2,\\ C &= \alpha^2\beta\,(2\Theta_1-\Theta_2-\Theta_3)-6\alpha d,\\ D &= \alpha\beta\,(\Theta_2+\Theta_3-2\Theta_4)-2d.\end{aligned}$$

Proof See Appendix.

Note that this is the only market structure for which we can write explicitly the functional form that characterizes the solution(s) for the amount of investment X^C. The third-degree polynomial in (16) can be written equivalently as follows:[16]

$$Ax^2+Bx+C=-\frac{D}{x}.$$

The number of solutions to (16) will depend on how many times the parabola (left hand side of the equation) and the hyperbola (right hand side) intersect. The second-degree polynomial $f(x)=Ax^2+Bx+C$ achieves its maximum at $x^*=-\frac{B}{2A}$. As A and B are strictly negative, the number of solutions to (16) can be characterized in terms of the signs of C and D as follows:

Case 1: If $C<0$ and $D<0$, then we have no solution.

Case 2: If either $C\leq 0$ and $D\geq 0$, or $C\geq 0$ and $D\geq 0$, then we have only one solution to (16).

Case 3: If $C>0$ and $D\leq 0$, then we can have two, one, or no solution.

[16]We are grateful to a reviewer for suggesting to add this analysis.

As C and D depend on the value functions through the Θs, we cannot sign them. Making the intuitive assumption that the value function is higher for lower cost, then we have $\Theta_2 + \Theta_3 - 2\Theta_4 > 0$. Indeed,

$$\begin{aligned}\Theta_2 + \Theta_3 - 2\Theta_4 &= 2(1-\delta)\left(w(c_A^-, c_B) - w\left(c_A, c_B\right)\right) + 2\gamma\delta\left(w\left(c_A, c_B\right) - w(c_A, c_B^+)\right) \\ &\quad +2\gamma(1-\delta)\left(w(c_A^-, c_B^-) - w(c_A^-, c_B)\right) + 2\delta w\left((c_A, c_B^+) - (c_A^+, c_B^+)\right).\end{aligned}$$

Therefore, a sufficient condition to have a unique solution is

$$D \geq 0 \Leftrightarrow \beta\frac{(\Theta_2 - \Theta_4) + (\Theta_3 - \Theta_4)}{2} \geq \frac{d}{\alpha}.$$

The above condition says that D is positive if the discounted average of $\Theta_2 - \Theta_4$ and $\Theta_3 - \Theta_4$ is at least equal to the ratio of marginal investment cost over effectiveness of investment. Moreover, since each Θ_l is an expected value for a particular combination of the firms' success of investment, $\Theta_2 - \Theta_4$ is the advantage of having innovation only in firm j relative to the case where none of the firms innovates. Similarly, $\Theta_3 - \Theta_4$ is the advantage of having innovation only in firm k relative to no innovation from either firm. Therefore, the expression above compares the average of gains when either of the two firms innovates relative to no innovation from either firm against the cost of innovation normalized by its effectiveness.[17]

The numerical approach to find the policy and value functions for the cartelization problem is similar to the one exposed in the previous section for the competition model. See the Online Appendix for details. We obtain the different environments for the different levels of spillovers following the parametrization given in Table 1 from Section 2.

4 Numerical Results

We compare the outcomes of three different collaboration environments: traditional R&D, spillovers, and RJV. For each of these environments we consider two market structures: competition and cartelization in the R&D decisions but keep the Bertrand competition assumption at the product level.

First we compute the model outcomes using the parameter values shown in Table 2. In Section 5 we show the results for a larger set of parameter values. Our motivation for the choice of these initial values is as follows. McGuire and Pakes (1994) use a discount factor $\beta = 0.925$, a market size $m = 5$, and a likelihood of success of investment $\alpha = 3$. Extensions of the Pakes-McGuire model have used similar values, including Besanko and Doraszelski (2004) who have used values for δ between 0 and 0.3, and $\alpha = 0.125$. In Doraszelski and Markovich (2007), θ takes on values between 0 and 20, and $\lambda = 1$. In summary, all of our parameter values fall within the intervals of parameter values used in the literature for the Pakes-McGuire model. In addition, our benchmark results consider two different values for the level of spillover: $\gamma = 0.3$ and $\gamma = 0.7$ denoted in the graphs as "low" and "high", respectively.

[17] In the numerical results, we always find a unique solution for $x > 0$.

Table 2: Parameter values

β	0.925
market size (m)	5
MC appreciation (δ)	0.1
investment cost (d)	1
max. MC (M)	18
min. MC	0
low γ	0.3
high γ	0.7
α	2.5
Utility function parameters	
λ	0.5
θ	4

4.1 Present Value of Profits

We begin by presenting the outcomes for the expected present value of profits (the value function) relative to the corresponding R&D version of each regime in percentage changes, see Figure 1. For the competition cases, the profits represent one single firm's profits, not the total industry profits.

Figure 1 shows that the result in Kamien et al. (1992) holds in the dynamic case as well: profits in the RJV cartelization environment are greater than under R&D. However, the gains can be very small if either of the two firms has access to low marginal costs technologies.

Claim 4.1. *(i) In the dynamic setting and for both the competition and the cartelization cases, the long-run present value profits in the RJV and the spillover cases are greater than the long-run present value profits in the R&D environment.*

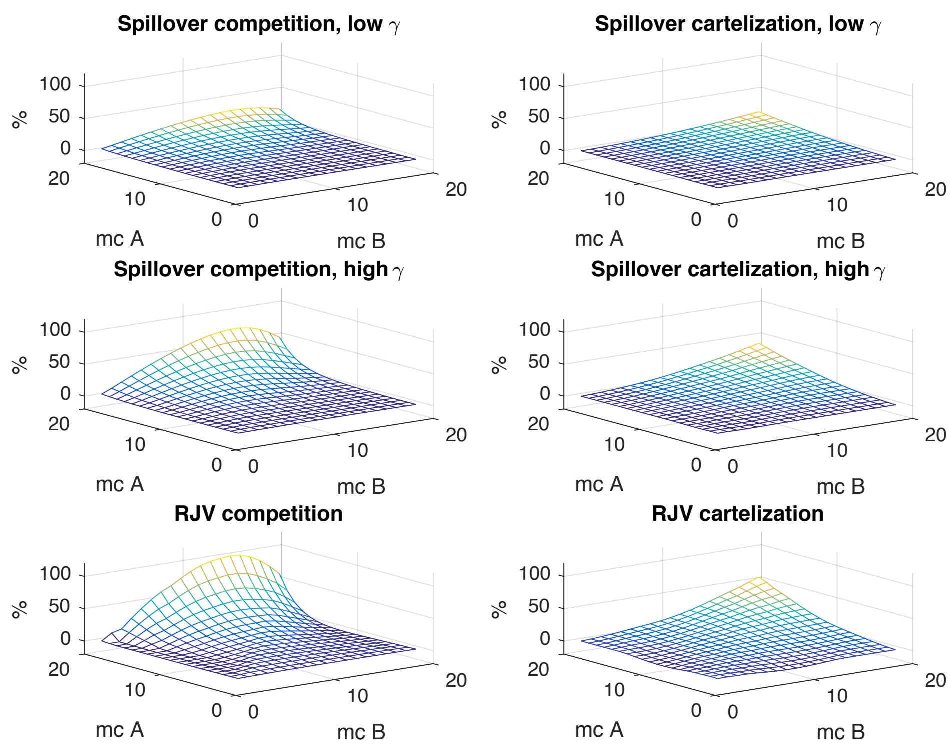
(ii) For the competition cases, these differences are larger when the asymmetry of the firms in their levels of marginal costs is larger. For the cartelization cases, these differences are larger for higher levels of marginal costs.

One comparison not covered in Kamien et al. (1992) is the one between the RJV and R&D competition environments. Here we provide evidence of larger gains (in percentage points) between these two regimes than in between the RJV and the R&D cartelization regimes. All these gains in discounted profits increase as the level of the spillover γ increases.

4.2 The Outside Good Market Share and Profits

Another way to compare the different environments is by looking at each period's outcomes. To do so, we begin at time $t = 0$ with a uniform distribution over the space of marginal costs. That is, we assign the same probability to each state (c_A, c_B) at $t = 0$. For each

Figure 1: Gains in expected discounted profits

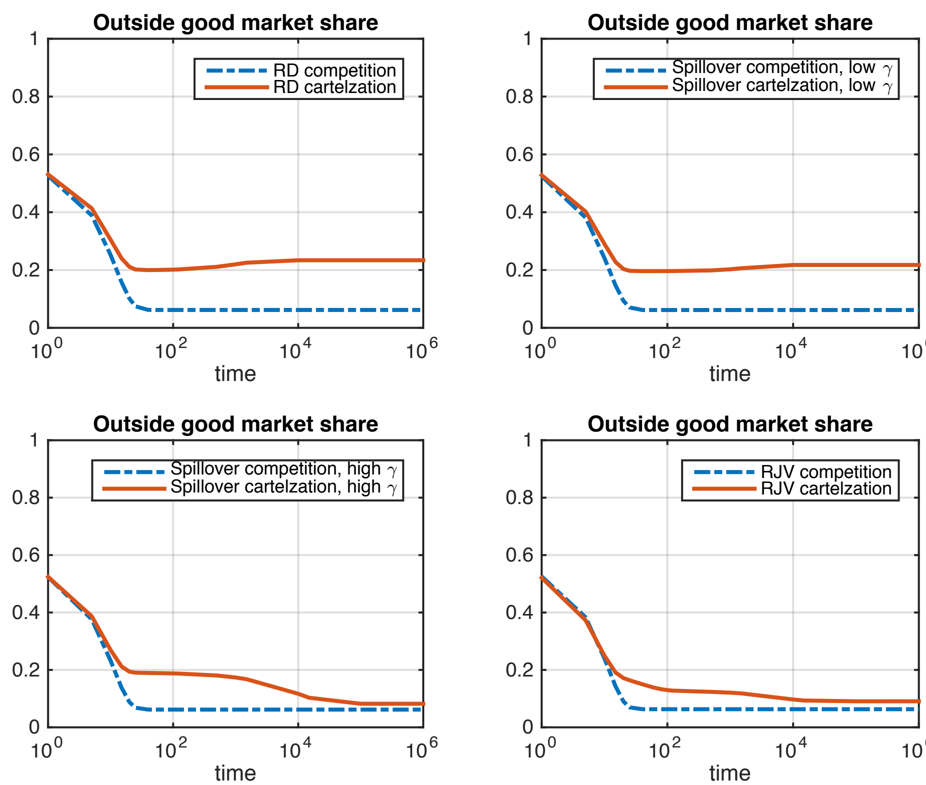


Notes: Each graph represents the difference in total expected discounted profits for the environment indicated in the title relative to the total expected discounted profits in the R&D environment (competition cases in the left column, cartelization cases in the right column) expressed in percentages. This is simply the difference between value functions.

$t > 0$, applying the converged policy function leads to a new distribution over the marginal cost space, we then use this transient distribution at each time period to compute expected values for the outside good market share, profits, consumer surplus, prices, and welfare. The derivation of the transition matrix used to generate the transient distribution to compute the expected values of outcomes is explained in the Online Appendix. For all the parameter values presented here, we have found convergence for the transition distribution. This limiting distribution is unique regardless of the initial conditions and thus the outcomes at convergence are not dependent on our particular choice of the uniform distribution at $t = 0$. Moreover, for the parameters used here, we find only one recurrent class in each of the limiting distributions for each environment.[18]

Figure 2 shows the evolution of the outside good market share for each environment and lead us to conclude the following.

Figure 2: Expected outside good market share



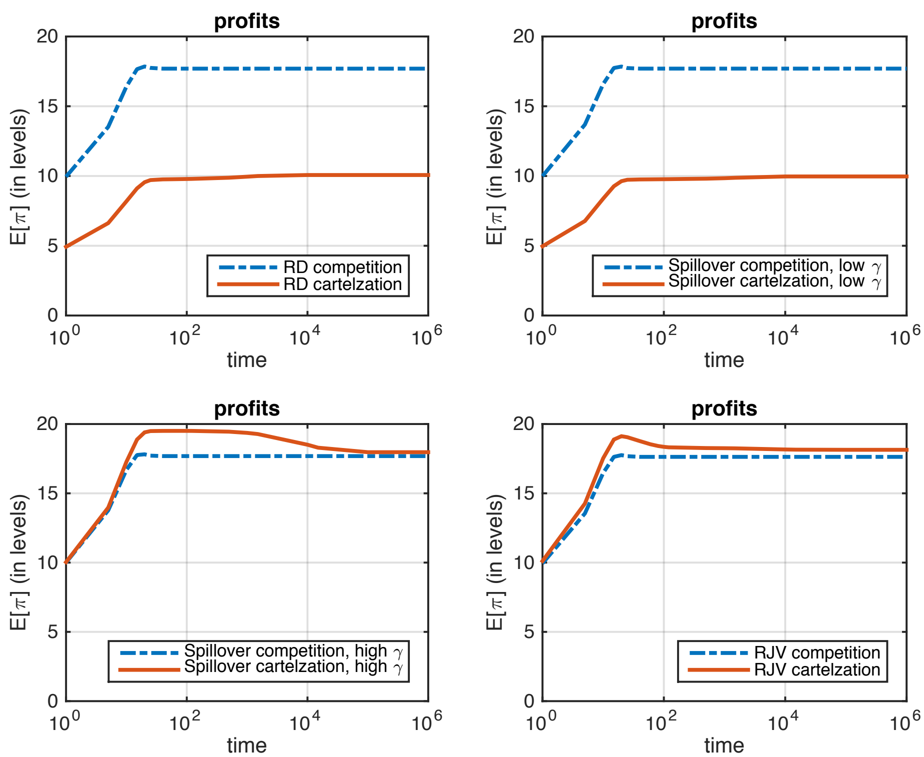
Notes: Each graph represents the expected value of the outside good market share using the transient distribution at each point in time.

[18]See Besanko and Doraszelski (2004) for a discussion of cases in which more than one recurrent class emerges. We present some transient distributions for different environments and different time periods in the Appendix.

Claim 4.2. *For a given level of information sharing (a fixed value of γ), the mean expected market share of the outside good is always lower in the competition regime than in the cartelization regime. Moreover, this market share is lower in the RJV cartelization case relative to the other cartelization environments.*

Thus an RJV environment allows for a faster and sustained expansion of the industry. In order to study the profits at each period we simply compute the integral of the static profits using the transient distribution. In Figure 3 we present the industry profits, where for the competition cases we present the sum of the two firms' profits. When there are no spillovers or their level is relatively low, industry profits are almost twice as large in the competition regime than in the cartelization regime. This implies that for a firm to prefer to get involved in an R&D cartel, enough spillovers should be guaranteed.

Figure 3: $E[\pi]$ per period



Notes: Initial distribution is a uniform densitiy over the MC-space. At each t, we compute the expected value using the transient distribution.

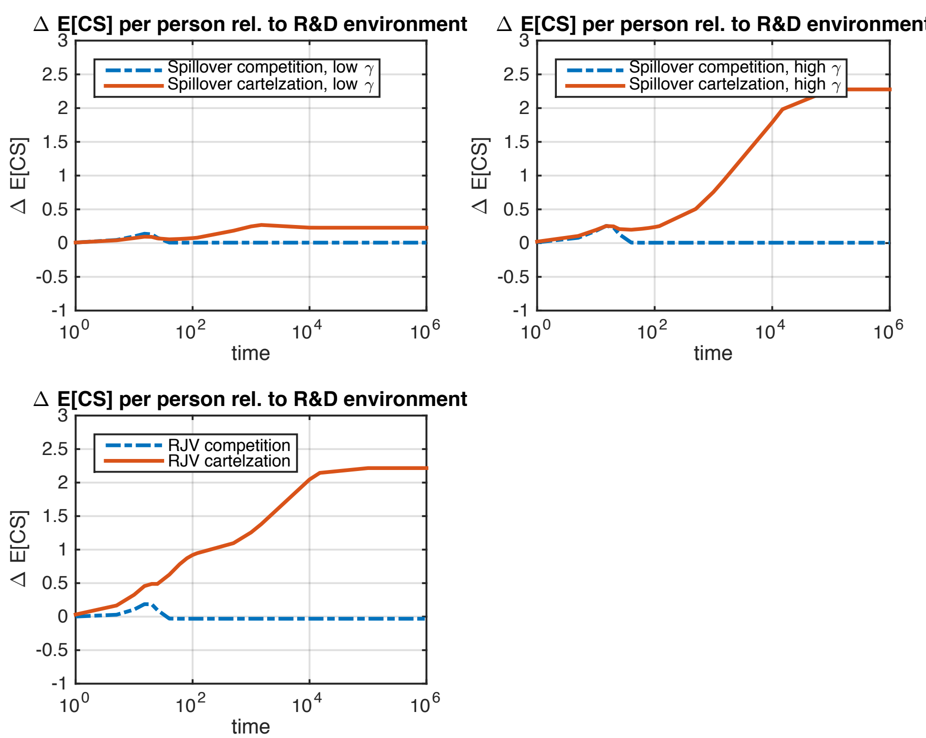
4.3 Consumer Surplus and Welfare

Consumer surplus in discrete choice models is obtained by the following formula known as the log-inclusive equation.[19] One consumer's surplus is the expected value of the utility function that leads to our logit demand forms,

$$CS = E[U] = \frac{1}{\lambda}\log(\exp(\theta - \lambda p_A) + \exp(\theta - \lambda p_B)) + C,$$

where C is a constant that reflects the fact that the utility function is defined up to an additive constant. The expected value is taken over the extreme value distributed random term of the utility function. λ is the marginal utility of income, which in our demand model is the negative of the price coefficient. The term inside the log is the denominator in the choice probability expression.

Figure 4: $\Delta E[CS]$ per period



Notes: Initial distribution is a uniform densitiy over the MC-space. At each t, we compute the expected value using the transient distribution.

One way to turn this expected value into an operational expression is to look at changes in consumer surplus when we switch from one model to another by keeping the utility function

[19] See Train (2009).

fixed. By taking the difference, we eliminate the constant C,

$$\Delta CS = \frac{1}{\lambda}\left(\log[\exp(\theta - \lambda p_A) + \exp(\theta - \lambda p_B)] - \log[\exp(\theta - \lambda p_{A0}) + \exp(\theta - \lambda p_{B0})]\right),$$

where p_{A0} and p_{B0} are the prices in the benchmark environment at a given state of the marginal cost space. Finally, this change in CS is multiplied by the size of the market. The outcome we present in the figures is the gains in consumer surplus from being in a given environment relative to the corresponding R&D regime. See Figure 4 for the results.

The increase in consumer surplus in the cartelization cases relative to the R&D benchmark can be explained by the decrease in expected prices. Figure 5 shows that prices drastically decrease for the cases of high spillover cartelization and RJV cartelization, relative to the expected price in the R&D cartelization case. This large drop in prices occurs because competition still exists at the product level regardless of the environment assumption for process innovation. Thanks to the high level in spillovers, marginal costs decrease, which translates into lower prices because of the competition effects. This in turn explains the large increases in consumer surplus.

Claim 4.3. *Consumer surplus gains increase with the level of the spillover. At each period, the mean gains in consumer surplus are larger in the RJV environment.*

In contrast to Kamien et al. (1992), we find that expected prices under RJV cartelization converge to higher levels than under RJV competition. However, over the first 20 or so time periods, our model finds the opposite, which would be in agreement with the previous findings in the literature.

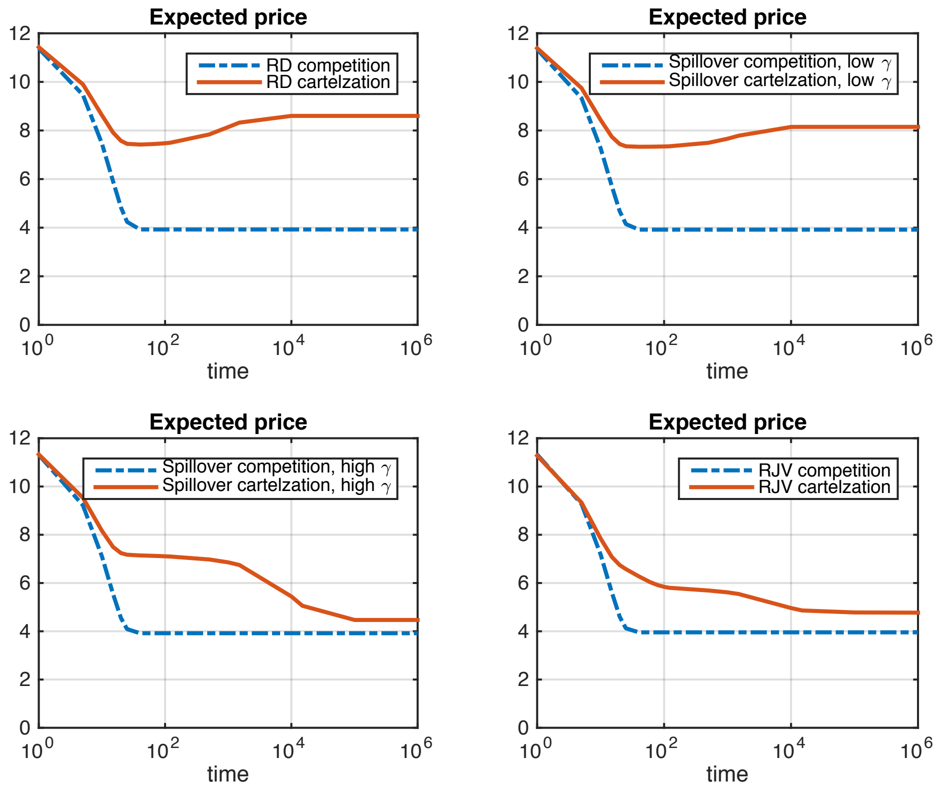
Claim 4.4. *(i) Contrary to the results for the 2-stage game from Section 2, expected prices under RJV cartelization are greater than in the RJV competition environment in the long-run.*
(ii) High levels of the spillover induce lower expected prices.

The intuition for this difference with the 2-stage model is as follows. In Kamien et al, their model can allow for perfect substitutes and lead to markups equal to zero. In our model, markups are always positive because the Bertrand equilibrium is characterized, in vector notation, by $\mathbf{p} = \mathbf{c} - \Omega^{-1}\mathbf{s}$. Where Ω is a diagonal matrix containing the derivatives of market shares with respect to prices. In the logit model they are $\Omega_{ii} = -\lambda s_i(1-s_i)$. Since in the logit model each product always has a positive market share, margins will never be zero even if the marginal costs are close to zero. This holds for the case of competition, therefore, for the cartelization case markups are even larger, which explains (i) in the Claim above. In the proof of results for the price competition case, Kamien et al needed to assume an upper bound on the substitution degree between the two goods: this bound induces the degree of differentiation needed to avoid a collapse of the markups in the cartelization case.[20]

The dynamics play an important role here as well as the stochastic processes. In the 2-stage game there is only one opportunity to reduce marginal costs, whereas in our model it may take several periods in order to obtain even a reduction of one unit if the draws for the

[20]See discussion after Proposition 1^* in Kamien et al. (1992).

Figure 5: $E[price]$



Notes: Initial distribution is a uniform densitiy over the MC-space. At each t, we compute the expected value using the transient distribution.

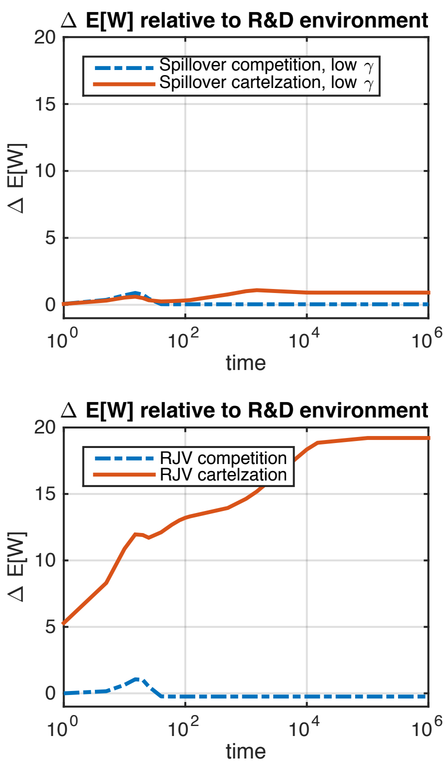
industry-wide shock are very persistent. Hence the importance of the level of the spillover, as reflected in the second part of the Claim above.

Finally, we present our results for welfare. The discussion about consumer surplus implies that even though we can compute the exact amount of profits at each period, we cannot compute total welfare, but only the change in the welfare relative to the benchmark. See Figure 6.

Claim 4.5. *The difference in expected long-run welfare between RJV cartelization and RJV competition is the largest among all the corresponding differences between cartelization and competition regimes.*

This has an important consequence for policymakers: if a form of collaboration is to be allowed, it is better to allow for full sharing of information and a cartel in the investment decisions. This is not only more acceptable from a normative point of view, as opposed to determining the value of collusion at the product level, but it can actually be quantified to yield higher welfare levels in the long-run.

Figure 6: $\Delta E[W]$ per period



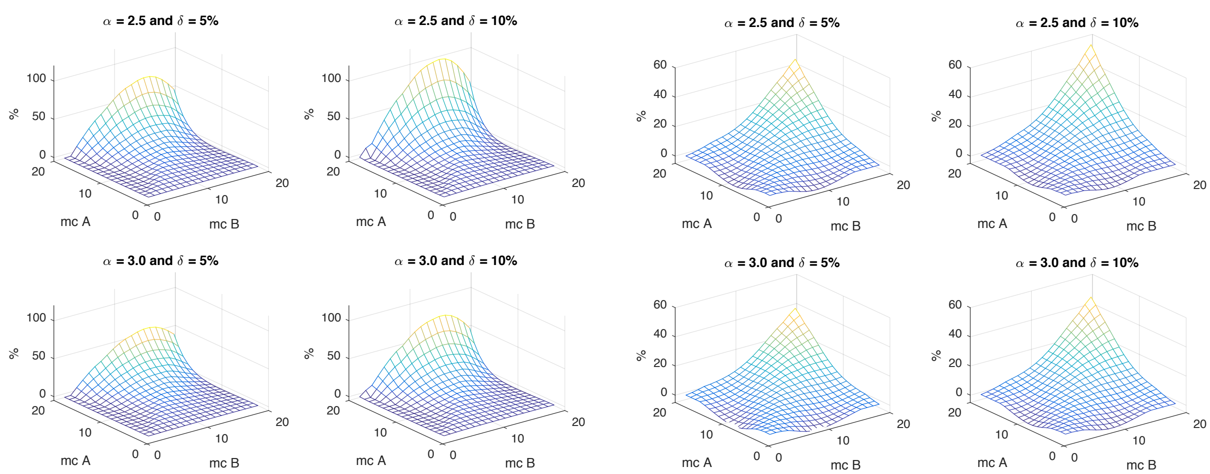
Notes: Initial distribution is a uniform densitiy over the MC-space. At each t, we compute the expected value using the transient distribution.

5 Sensitivity Analysis

We now turn to the analysis of the impact of parameter values on outcomes. Our goal is to show that the results from the previous section hold for an economically-relevant range of parameter values.

On the role of γ. This parameter represents the degree of the spillover of information sharing. The higher its value the higher the spillover. Its upper bound, $\gamma = 1$, represents the RJV environment. In Section 4 we have analyzed the impact of this parameter on outcomes for three other values. All the cases of traditional R&D are associated with a value of $\gamma = 0$. In our main results we have compared value functions, outside good market shares, prices, consumer surplus, prices, and welfare for low and high levels of the spillover ($\gamma = 0.3$ and $\gamma = 0.7$, respectively). Those results lead us to conclude that holding the other parameter values fixed, a higher level of the spillover causes a higher advantage in total expected discounted profits for firms relative to the R&D benchmark. These higher profits are associated with higher inside good market shares because prices drop in the long term when γ increases. Because competition on prices exists at each time period, a higher spillover reduces marginal costs for both firms and accelerates the homogenization of their cost structure. This in turn amplifies the level of competition, which in turn makes prices fall.

Figure 7: Effects on the value functions.

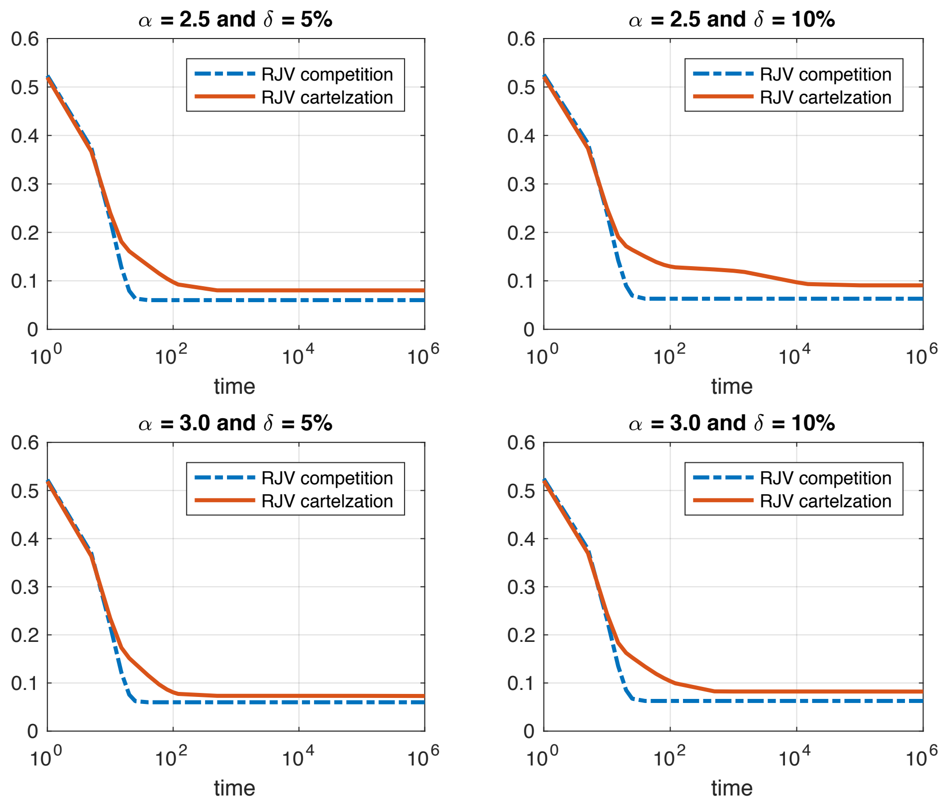


(a) Competition (firm A). (b) Cartelization.

Notes: Each graph represents the difference of the value function for the environment indicated in the title relative to the value function in the R&D environment, expressed in percentages.

On the role of α and δ. We now concentrate on the impact of different values for the likelihood of success of investment α and the rate of appreciation of marginal costs δ. Specifically, we assess the validity of our claims in Section 4 for different combinations of these parameters. In Claim 1 we compared the converged value functions for different values of the spillover against the converged value function when there is no spillover. We found that the value function is greater when there is a positive level of spillover. Here we fix the value of the spillover to $\gamma = 1$ and solve for the equilibrium at different levels of α and δ. The motivation for this is to explore the robustness of our results at the maximum impact of the spillover since it is intuitively clear that for weaker levels of the spillover similar results arise except that the differences between value functions are smaller.[21]

Figure 8: Expected outside good market share



Notes: Each graph represents the expected value of the outside good market share for different values of α and δ using the transient distribution at each point in time.

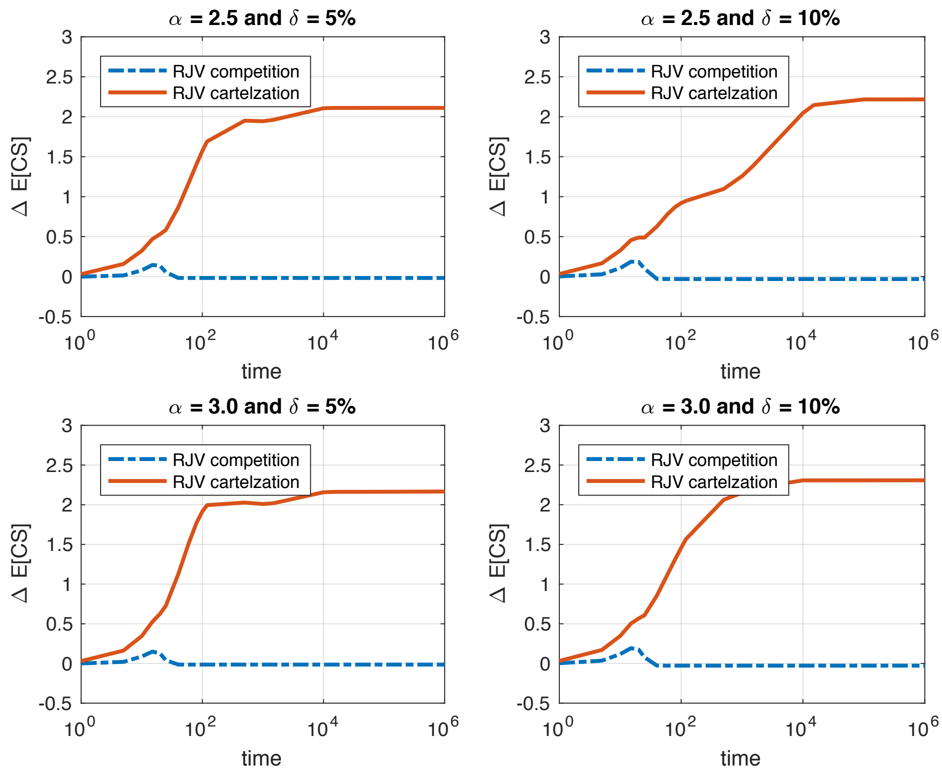
Figure 7 shows differences of the value function under RJV and under R&D for four different combinations of values for α and δ. In all cases, competition and cartelization, RJV yields higher discounted present values for profits for all marginal cost combinations. This is consistent with Claim 1. Moreover, these differences increase with the appreciation rate because a full spillover is more valuable in an environment where costs can increase with a higher probability. Similarly, these differences are larger when α is smaller because if the probability of success of investment is smaller, having access to the full spillover has more value relative to the R&D environment.

In Figure 8 we show the effect on outside good market shares between competition and cartelization for the RJV environment. In all cases shown, the outside good market share

[21] Results for other parameter values and the code are available upon request.

is higher under cartelization: the full cooperation of firms allows for larger market shares for the two firms. This is consistent with Claim 2. As suggested by the figures, the rate of growth of the inside good market share is lower when the probability of success of investment is lower and the rate of appreciation of cost is higher.

Figure 9: $\Delta E[CS]$ per period

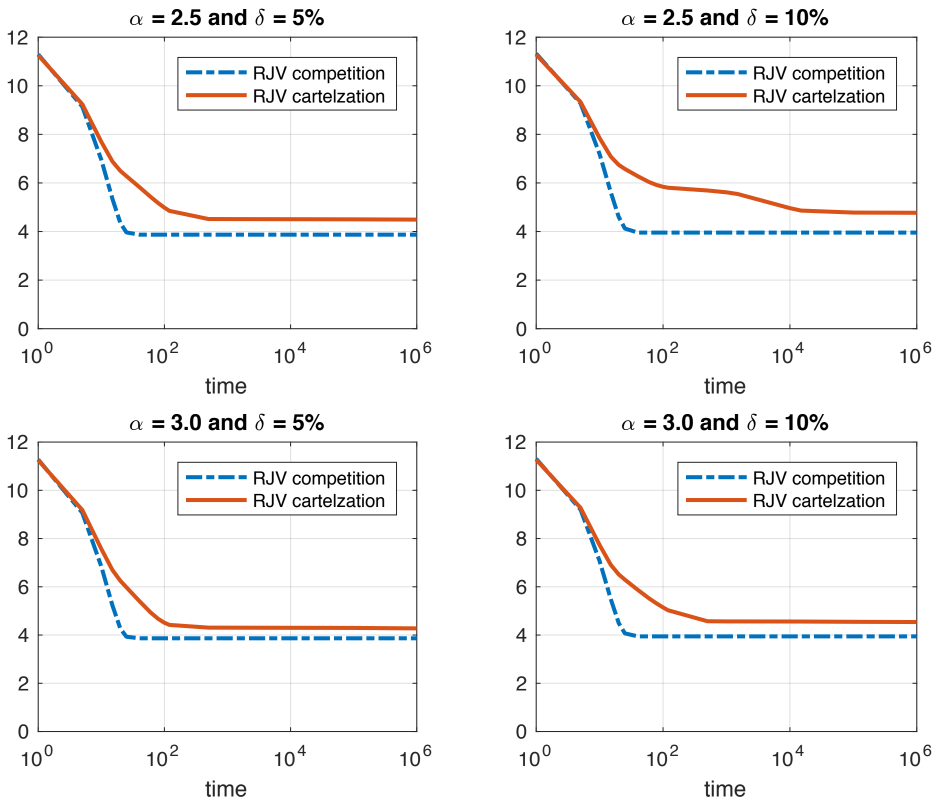


Notes: Each panel shows results for a different combination of values for α and δ. Initial distribution is a uniform densitiy over the MC-space. At each t, we compute the expected value using the transient distribution.

We conduct a similar analysis for the expected consumer surplus and the expected level of prices over time. This is shown in Figures 9 and 10. In both cases, our results are consistent with Claims 3 and 4, respectively. Since we keep the level of product competition the same across the different environments, we can observe the effect on consumer surplus due only to an improvement in the likelihood of success of investment. Although the long-term levels are similar, the transition paths are not: the long-term level is attained faster when α is higher and the rate of appreciation lower. The effect on prices is less pronounced but nonetheless consistent across the different combinations of parameters.

Finally, since prices, market shares, and consumer surplus respond similarly for a wide range of parameters including the benchmark parameters, our Claim 5 regarding welfare holds as well. Given that the primarily goal of this paper is to study the effects of the different cooperation environments on the market structure, we focused on the most relevant

Figure 10: $E[price]$



Notes: Each panel shows results for a different combination of values for α and δ. Initial distribution is a uniform densitiy over the MC-space. At each t, we compute the expected value using the transient distribution.

aspects of the supply parameters.

6 Conclusion

The effects of different collaboration environments for R&D can vary drastically. If regulations are to put in place collaboration incentives among firms in their research endeavors, it is important to understand the market consequences of such incentives.

We have found evidence of positive effects on welfare from allowing collaboration and information sharing at the process innovation level. While keeping a competitive environment at the product level, we could measure the effects due only to collusive strategies and spillovers at the R&D level. This speaks to Schumpeter's hypothesis: the higher the concentration the higher the innovation, in that if concentration is thought to be at the process innovation level, then the hypothesis is true.

We allowed for a stochastic dynamic process for the success of investment. This is crucial to understand the trade-off between better innovation processes (higher likelihood of success) and the resources needed to be invested if a certain goal is to be achieved at a pre-determined time. This is an important characteristic for future research: if these types of models are taken to the data, a deterministic model would not be able to predict outcomes with the same flexibility as a stochastic model.

Another issue for future research is the duplicate nature of investments and its consequences for outcomes. In our model, even if both firms succeed in their corresponding labs and there is full information sharing, marginal costs decrease by one unit for each firm. One could think of a situation of non-duplicate research, in which if the two labs succeed, then marginal costs could decrease by more than one unit in the same period.

Our framework nonetheless, offers a unifying setting to analyze different environments of information sharing and collaboration. We believe it can guide policymakers and future research in expanding our understanding of the incentives related to process innovation and collusion.

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Appendix

Proofs

Proof for Proposition 3.1.

For $j = 1, 2$, given x_k, firm j's first-order condition is

$$\begin{aligned}
&-d + \beta\left(\frac{\partial\phi(x_j)}{\partial x_j}\phi(x_k)\right)\cdot\Delta_{j,1}\\
&+\beta\left(\frac{\partial\phi(x_j)}{\partial x_j}(1-\phi(x_k))\right)\cdot\Delta_{j,2}\\
&-\beta\left(\frac{\partial\phi(x_j)}{\partial x_j}\phi(x_k)\right)\cdot\Delta_{j,3}\\
&-\beta\left(\frac{\partial\phi(x_j)}{\partial x_j}(1-\phi(x_k))\right)\cdot\Delta_{j,4} = 0
\end{aligned}$$

where $\partial\phi(x_j)/\partial x_j = \alpha/\left(1+\alpha x_j\right)^2$ and $\phi(x_k) = \alpha x_k/(1+\alpha x_k)$. The second-order condition

$$\begin{aligned}
&\beta\left(\frac{\partial^2\phi(x_j)}{\partial x_j^2}\phi(x_k)\right)\cdot\Delta_{j,1}\\
&+\beta\left(\frac{\partial^2\phi(x_j)}{\partial x_j^2}(1-\phi(x_k))\right)\cdot\Delta_{j,2}\\
&-\beta\left(\frac{\partial^2\phi(x_j)}{\partial x_j^2}\phi(x_k)\right)\cdot\Delta_{j,3}\\
&-\beta\left(\frac{\partial^2\phi(x_j)}{\partial x_j^2}(1-\phi(x_k))\right)\cdot\Delta_{j,4} < 0
\end{aligned}$$

$\frac{\partial^2\phi(x_j)}{\partial x_j^2} = \frac{-2\alpha^2}{(1+\alpha x_j)^3}$ is satisfied when $\frac{\alpha x_k(\Delta_{j,1}-\Delta_{j,3})}{1+\alpha x_k} + \frac{(\Delta_{j,2}-\Delta_{j,4})}{1+\alpha x_k} \geq 0$, which yields

$$R\left(x_k\right) = \max\left\{-\frac{1}{\alpha} + \sqrt{\frac{\beta}{\alpha d}}\sqrt{\frac{\alpha x_k\left(\Delta_{j,1}-\Delta_{j,3}\right)}{1+\alpha x_k} + \frac{\Delta_{j,2}-\Delta_{j,4}}{1+\alpha x_k}}, 0\right\}$$

where the max operator accounts for the usual constraint $x_j \geq 0$. Next, if $\frac{\alpha x_k(\Delta_{j,1}-\Delta_{j,3})}{1+\alpha x_k} + \frac{(\Delta_{j,2}-\Delta_{j,4})}{1+\alpha x_k} < 0$, the second-order condition is not satisfied and thus $R\left(x_k\right) = 0$.

Proof for Proposition 3.2.

The first-order condition is

$$-2d + 2\beta\phi(x)\phi'(x)\cdot\Theta_1 + \beta\left(1-2\phi(x)\right)\phi'(x)\cdot\left(\Theta_2+\Theta_3\right) - 2\beta(1-\phi(x))\phi'(x)\cdot\Theta_4 = 0$$

where

$$\begin{aligned}\phi(x) &= \frac{\alpha x}{1+\alpha x} \\ \phi'(x) &= \frac{\alpha}{(1+\alpha x)^2}.\end{aligned}$$

The first-order condition simplifies to

$$-2d(1+\alpha x)^3 + 2\beta\alpha^2 x \cdot (\Theta_1 + \Theta_4 - (\Theta_2 + \Theta_3)) + \alpha\beta(1+\alpha x)(\Theta_2 + \Theta_3 - 2\Theta_4) = 0$$

or

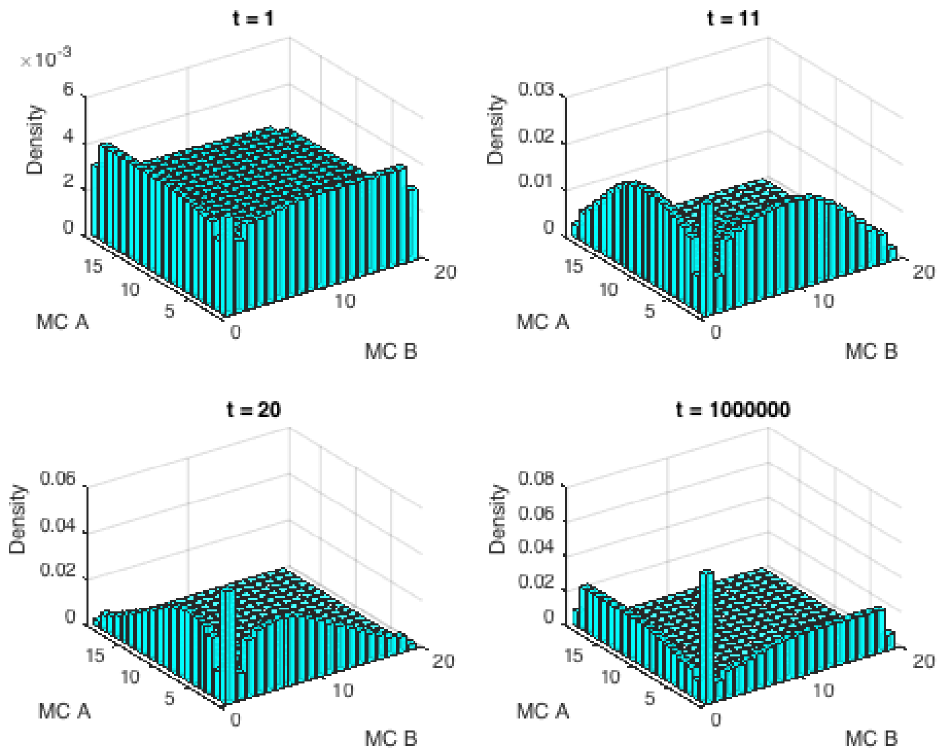
$$Ax^3 + Bx^2 + Cx + D = 0$$

where

$$\begin{aligned} A &= -2d\alpha^3, \\ B &= -6d\alpha^2, \\ C &= \alpha^2\beta(2\Theta_1 - \Theta_2 - \Theta_3) - 6\alpha d, \\ D &= \alpha\beta(\Theta_2 + \Theta_3 - 2\Theta_4) - 2d. \end{aligned}$$

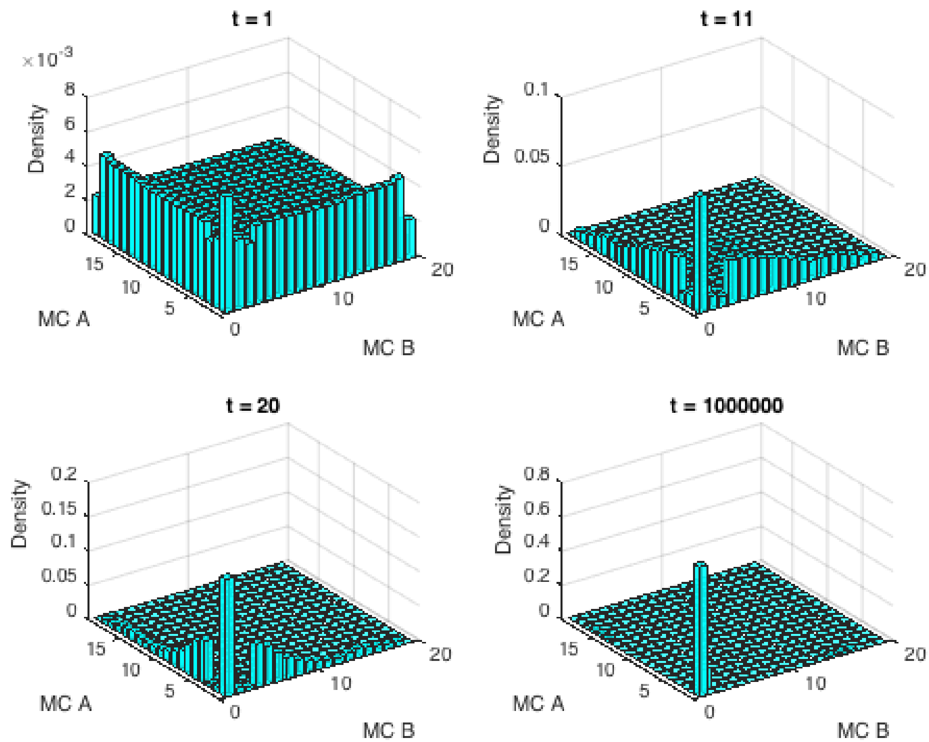
Transient Distributions

Figure 11: Transient distribution: R&D cartelization



Notes: Initial distribution is a uniform densitiy over the MC-space.

Figure 12: Transient distribution: RJV cartelization



Notes: Initial distribution is a uniform densitiy over the MC-space.

Online Appendix (not for publication)

Numerical Approach for Cartelization

For $\tau = 0$, for all (c_A, c_B), $X^0(c_A, c_B) = 0$ and

$$W^0(c_A, c_B) = \Pi(c_A, c_B) + \Pi(c_B, c_A).$$

For $\tau > 1$, the investment strategy $X^\tau(c_{1,}c_B)$ is the solution to the polynomial of degree 3 defined above as long as $X^\tau(c_{1,}c_B) \geq 0$ and the second-order condition is satisfied, i.e.,

$$\begin{aligned}
&2\beta\phi'(x)\phi'(x)\cdot\Theta_1^{\tau-1} + 2\beta\phi(x)\phi''(x)\cdot\Theta_1^{\tau-1} \\
&-2\beta\phi'(x)\phi'(x)\cdot(\Theta_2^{\tau-1}+\Theta_3^{\tau-1}) + \beta\left(1-2\phi(x)\right)\phi''(x)\cdot(\Theta_2^{\tau-1}+\Theta_3^{\tau-1}) \\
&+2\beta\phi(x)'\phi'(x)\cdot\Theta_4^{\tau-1} - 2\beta(1-\phi(x))\phi''(x)\cdot\Theta_4^{\tau-1} < 0
\end{aligned}$$

where

$$\phi''(x) = -\frac{2\alpha^2}{(1+\alpha x)^3}.$$

If not, then $X^\tau(c_{1,}c_B) = 0$. Here,

$$\begin{aligned}
\Theta_1^{\tau-1} \equiv\ & (1-\gamma)\left[\delta W^{\tau-1}(c_j, c_k) + (1-\delta)W^{\tau-1}(c_j^-, c_k^-)\right] \\
& +\gamma\psi[\delta W^{\tau-1}\left(c_j^-, c_k^-\right) + (1-\delta)W^{\tau-1}(c_j^{--}, c_k^{--})] \\
& +\gamma(1-\psi)[\delta W^{\tau-1}(c_j, c_k) + (1-\delta)W^{\tau-1}(c_j^-, c_k^-)],
\end{aligned}$$

$$\begin{aligned}
\Theta_2^{\tau-1} \equiv\ & (1-\gamma)[\delta W^{\tau-1}(c_j, c_k^+) + (1-\delta)W^{\tau-1}(c_j^-, c_k)] \\
& +\gamma[\delta W^{\tau-1}(c_j, c_k) + (1-\delta)W^{\tau-1}(c_j^-, c_k^-)],
\end{aligned}$$

$$\begin{aligned}
\Theta_3^{\tau-1} \equiv\ & (1-\gamma)[\delta W^{\tau-1}(c_j^+, c_k) + (1-\delta)W^{\tau-1}(c_j, c_k^-)] \\
& +\gamma[\delta W^{\tau-1}(c_j, c_k) + (1-\delta)W^{\tau-1}(c_j^-, c_k^-)],
\end{aligned}$$

and

$$\Theta_4^{\tau-1} \equiv \delta W^{\tau-1}(c_j^+, c_k^+) + (1-\delta)W^{\tau-1}(c_j, c_k)$$

depends on the value function computed at the $(\tau - 1)$ iteration. Finally,

$$\begin{aligned}
W^\tau(c_A, c_B) =\ & \Pi(c_A, c_B) + \Pi(c_B, c_A) - 2dX^\tau(c_{1,}c_B) \\
& +\beta\phi(X^\tau(c_{1,}c_B))\phi(X^\tau(c_{1,}c_B))\cdot\Theta_1^{\tau-1} \\
& +\beta\phi(X^\tau(c_{1,}c_B))(1-\phi(X^\tau(c_{1,}c_B))))\cdot\Theta_2^{\tau-1} \\
& +\beta(1-\phi(X^\tau(c_{1,}c_B)))\phi(X^\tau(c_{1,}c_B))\cdot\Theta_3^{\tau-1} \\
& +\beta(1-\phi(X^\tau(c_{1,}c_B)))(1-\phi(X^\tau(c_{1,}c_B)))\cdot\Theta_4^{\tau-1}.
\end{aligned}$$

The iteration continues until convergence in $X(c_A, c_B)$ and $W(c_A, c_B)$ is reached for all (c_A, c_B).

Transition Probability Matrix

In this appendix, we describe the transition matrix for each case. In general, let $c_j \in \{0, 1, 2, ..., M\}$ such that $M \geq 2$.

R&D

In the case of R&D competition or cartelization, for $j, k \in \{A, B\}, j \neq k$, the stochastic process for cost is

$$\tilde{c}_j'|c_j = \min\{\max\{c_j + \tilde{\tau}_j + \tilde{\eta}, 0\}, M\}$$

where $\tau_j \in \{-1, 0\}$ such that $\Pr[\tilde{\tau}_j = -1] = \phi(c_j, c_k) = \frac{\alpha X(c_j, c_k)}{1+\alpha X(c_j, c_k)}$ and $\eta \in \{0, 1\}$ such that $\Pr[\tilde{\eta} = 1] = \delta$. Here, $X(c_j, c_k)$ is firm j's policy function corresponding either to R&D competition or R&D cartelization. We now define each element $\Pr[(c_A', c_B') \,|\, (c_A, c_B)]$ of the transition matrix conditional on (c_A, c_B). When not defined, the element is equal to zero. The term Ω is the probability of $(c_A', c_B') \neq (c_A, c_B)$.

1. Suppose that (c_A, c_B) is such that $c_A, c_B \notin \{0, M\}$. Then,

$$\begin{aligned}
\Pr[(c_A, c_B) \,|\, (c_A, c_B)] &= 1 - \Omega, \\
\Pr[(c_A - 1, c_B) \,|\, (c_A, c_B)] &= (1-\delta)\phi(c_A, c_B)(1 - \phi(c_B, c_A)), \\
\Pr[(c_A + 1, c_B) \,|\, (c_A, c_B)] &= \delta(1 - \phi(c_A, c_B))\phi(c_B, c_A), \\
\Pr[(c_A, c_B + 1) \,|\, (c_A, c_B)] &= \delta\phi(c_A, c_B)(1 - \phi(c_B, c_A)), \\
\Pr[(c_A + 1, c_B + 1) \,|\, (c_A, c_B)] &= \delta(1 - \phi(c_A, c_B))(1 - \phi(c_B, c_A)), \\
\Pr[(c_A, c_B - 1) \,|\, (c_A, c_B)] &= (1-\delta)(1 - \phi(c_A, c_B))\phi(c_B, c_A), \\
\Pr[(c_A - 1, c_B - 1) \,|\, (c_A, c_B)] &= (1-\delta)\phi(c_A, c_B)\phi(c_B, c_A).
\end{aligned}$$

2. Suppose that $(c_A, c_B) = (0, 0)$. Then,

$$\begin{aligned}
\Pr[(0, 0) \,|(0, 0)] &= 1 - \Omega, \\
\Pr[(1, 0) \,|(0, 0)] &= \delta(1 - \phi(0, 0))\phi(0, 0), \\
\Pr[(0, 1) \,|(0, 0)] &= \delta\phi(0, 0)(1 - \phi(0, 0)), \\
\Pr[(1, 1) \,|(0, 0)] &= \delta(1 - \phi(0, 0))(1 - \phi(0, 0)).
\end{aligned}$$

3. Suppose that $(c_A, c_B) = (M, M)$. Then,

$$\begin{aligned}
\Pr[(M, M) \,|\, (M, M)] &= 1 - \Omega, \\
\Pr[(M - 1, M) \,|\, (M, M)] &= (1-\delta)\phi(M, M)(1 - \phi(M, M)), \\
\Pr[(M, M - 1) \,|\, (M, M)] &= (1-\delta)(1 - \phi(M, M))\phi(M, M), \\
\Pr[(M - 1, M - 1) \,|\, (M, M)] &= (1-\delta)(1 - \phi(M, M))(1 - \phi(M, M)).
\end{aligned}$$

4. Suppose that $(c_A, c_B) = (0, M)$. Then,

$$\begin{aligned}
\Pr[(0,M)\,|\,(0,M)] &= 1-\Omega, \\
\Pr[(1,M)\,|\,(0,M)] &= \delta(1-\phi\,(0,M)), \\
\Pr[(0,M-1)\,|\,(0,M)] &= (1-\delta)\phi\,(0,M)\,.
\end{aligned}$$

5. Suppose that $(c_A, c_B) = (M, 0)$. Then,

$$\begin{aligned}
\Pr[(M,0)\,|\,(M,0)] &= 1-\Omega, \\
\Pr[(M,1)\,|\,(M,0)] &= \delta(1-\phi\,(M,0)\,, \\
\Pr[(M-1,0)\,|\,(M,0)] &= (1-\delta)\phi\,(0,M)\,.
\end{aligned}$$

6. Suppose that (c_A, c_B) is such that $c_A = 0$ and $c_B \notin \{0, M\}$. Then,

$$\begin{aligned}
\Pr[(0,c_B)\,|\,(0,c_B)] &= 1-\Omega \\
\Pr[(1,c_B)\,|\,(0,c_B)] &= \delta\,(1-\phi\,(0,c_B))\,\phi\,(c_B,0)\,, \\
\Pr[(0,c_B-1)\,|\,(0,c_B)] &= (1-\delta)\phi\,(c_B,0)\,, \\
\Pr[(0,c_B+1)\,|\,(0,c_B)] &= \delta\phi\,(0,c_B)\,(1-\phi\,(c_B,0)), \\
\Pr[(1,c_B+1)\,|\,(0,c_B)] &= \delta(1-\phi\,(0,c_B))(1-\phi\,(c_B,0)).
\end{aligned}$$

7. Suppose that (c_A, c_B) is such that $c_A \notin \{0, M\}$ and $c_B = 0$. Then,

$$\begin{aligned}
\Pr[(c_A,0)\,|\,(c_A,0)] &= 1-\Omega, \\
\Pr[(c_A,1)\,|\,(c_A,0)] &= \delta\phi\,(c_A,0)\,(1-\phi\,(0,c_A)), \\
\Pr[(c_A-1,0)\,|\,(c_A,0)] &= (1-\delta)\phi\,(c_A,0)\,, \\
\Pr[(c_A+1,0)\,|\,(c_A,0)] &= \delta\,(1-\phi\,(c_A,0))\,\phi\,(0,c_A)\,, \\
\Pr[(c_A+1,1)\,|\,(c_A,0)] &= \delta(1-\phi\,(0,c_A))(1-\phi\,(c_A,0)).
\end{aligned}$$

8. Suppose that (c_A, c_B) is such that $c_A = M$ and $c_B \notin \{0, M\}$. Then,

$$\begin{aligned}
\Pr[(M,c_B)\,|\,(M,c_B)] &= 1-\Omega, \\
\Pr[(M-1,c_B)\,|\,(M,c_B)] &= (1-\delta)\phi\,(M,c_B)\,(1-\phi\,(c_B,M)), \\
\Pr[(M,c_B-1)\,|\,(M,c_B)] &= (1-\delta)\phi\,(c_B,M)\,(1-\phi\,(M,c_B)), \\
\Pr[(M-1,c_B-1)\,|\,(M,c_B)] &= (1-\delta)\phi\,(M,c_B)\,\phi\,(c_B,M)\,, \\
\Pr[(M,c_B+1)\,|\,(M,c_B)] &= \delta(1-\phi\,(c_B,M)).
\end{aligned}$$

9. Suppose that (c_A, c_B) is such that $c_A \notin \{0, M\}$ and $c_B = M$. Then,

$$\begin{aligned}
\Pr[(M,c_A)\,|\,(c_A,M)] &= 1-\Omega, \\
\Pr[(c_A,M-1)\,|\,(c_A,M)] &= (1-\delta)\phi\,(M,c_A)\,(1-\phi\,(c_A,M)), \\
\Pr[(c_A-1,M)\,|\,(c_A,M)] &= (1-\delta)(1-\phi\,(M,c_A))\phi\,(c_A,M)\,, \\
\Pr[(c_A-1,M-1)\,|\,(c_A,M)] &= (1-\delta)\phi\,(M,c_A)\,\phi\,(c_A,M)\,, \\
\Pr[(c_A+1,M)\,|\,(c_A,M)] &= \delta(1-\phi\,(c_A,M)).
\end{aligned}$$

RJV-Duplicate

In the case of RJV-duplicate competition or cartelization, for $j,k \in \{A,B\}, j \neq k$, the stochastic process for cost is

$$\tilde{c}_j'|c_j = \min\{\max\{c_j + \min\{\tilde{\tau}_j, \tilde{\tau}_k\} + \tilde{\eta}, 0\}, M\}$$

where $\tau_j \in \{-1, 0\}$ such that $\Pr[\tilde{\tau}_j = -1] = \phi(c_j, c_k) = \frac{\alpha X(c_j,c_k)}{1+\alpha X(c_j,c_k)}$ and $\eta \in \{0,1\}$ such that $\Pr[\tilde{\eta} = 1] = \delta$. Here, $X(c_j, c_k)$ is firm j's policy function corresponding either to RJV-duplicate competition or cartelization. We now define each element $\Pr[(c_A', c_B') \,|\, (c_A, c_B)]$ of the transition matrix conditional on (c_A, c_B). When not defined, the element is equal to zero. The term Ω is the probability of $(c_A', c_B') \neq (c_A, c_B)$.

1. Suppose that (c_A, c_B) is such that $c_A \notin \{0, M\}$ and $c_B \notin \{0, M\}$. Then,

$$\begin{aligned}
\Pr[(c_A, c_B) \,|\, (c_A, c_B)] &= 1 - \Omega, \\
\Pr[(c_A - 1, c_B - 1) \,|\, (c_A, c_B)] &= \phi(c_A, c_B)\,\phi(c_B, c_A) \\
&\quad +(1-\delta)\phi(c_A, c_B)\,(1 - \phi(c_B, c_A)) \\
&\quad +(1-\delta)\phi(c_B, c_A)\,(1 - \phi(c_A, c_B)), \\
\Pr[(c_A + 1, c_B + 1) \,|\, (c_A, c_B)] &= \delta\,(1 - \phi(c_A, c_B))\,(1 - \phi(c_B, c_A)).
\end{aligned}$$

2. Suppose that $(c_A, c_B) = (0, 0)$. Then,

$$\begin{aligned}
\Pr[(0,0) \,|(0,0)] &= 1 - \Omega, \\
\Pr[(1,1) \,|(0,0)] &= \delta\,(1 - \phi(0,0))\,(1 - \phi(0,0)).
\end{aligned}$$

3. Suppose that $(c_A, c_B) = (M, M)$. Then,

$$\begin{aligned}
\Pr[(M,M) \,|\, (M,M)] &= 1 - \Omega, \\
\Pr[(M-1, M-1) \,|\, (M,M)] &= \phi(M,M)\,\phi(M,M) \\
&\quad +2(1-\delta)\phi(M,M)\,(1 - \phi(M,M)).
\end{aligned}$$

4. Suppose that $(c_A, c_B) = (0, M)$. Then,

$$\begin{aligned}
\Pr[(0,M) \,|\, (0,M)] &= 1 - \Omega, \\
\Pr[(0, M-1) \,|\, (0,M)] &= \phi(0,M)\,\phi(M,0) \\
&\quad +(1-\delta)\phi(0,M)\,(1 - \phi(M,0)) \\
&\quad +(1-\delta)\phi(M,0)\,(1 - \phi(0,M)), \\
\Pr[(1,M) \,|\, (0,M)] &= \delta\,(1 - \phi(0,M))\,(1 - \phi(M,0)).
\end{aligned}$$

5. Suppose that $(c_A, c_B) = (M, 0)$. Then,

$$\begin{aligned}
\Pr[(M,0)\,|\,(M,0)] &= 1-\Omega, \\
\Pr[(M-1,0)\,|\,(M,0)] &= \phi\,(M,0)\,\phi\,(0,M) \\
&\quad +(1-\delta)\phi\,(M,0)\,(1-\phi\,(0,M)) \\
&\quad +(1-\delta)\phi\,(0,M)\,(1-\phi\,(M,0))\,, \\
\Pr[(M,1)\,|\,(M,0)] &= \delta\,(1-\phi\,(M,0))\,(1-\phi\,(0,M))\,.
\end{aligned}$$

6. Suppose that (c_A, c_B) is such that $c_A = 0$ and $c_B \notin \{0, M\}$. Then,

$$\begin{aligned}
\Pr[(0,c_B)\,|\,(0,c_B)] &= 1-\Omega, \\
\Pr[(0,c_B-1)\,|\,(0,c_B)] &= \phi\,(0,c_B)\,\phi\,(c_B,0) \\
&\quad +(1-\delta)\phi\,(0,c_B)\,(1-\phi\,(c_B,0)) \\
&\quad +(1-\delta)\phi\,(c_B,0)\,(1-\phi\,(0,c_B)) \\
\Pr[(1,c_B+1)\,|\,(0,c_B)] &= \delta\,(1-\phi\,(0,c_B))\,(1-\phi\,(c_B,0))\,.
\end{aligned}$$

7. Suppose that (c_A, c_B) is such that $c_A \notin \{0, M\}$ and $c_B = 0$. Then,

$$\begin{aligned}
\Pr[(c_A,0)\,|\,(c_A,0)] &= 1-\Omega, \\
\Pr[(c_A-1,0)\,|\,(c_A,0)] &= \phi\,(c_A,0)\,\phi(0,c_A) \\
&\quad +(1-\delta)\phi\,(c_A,0)\,(1-\phi(0,c_A)) \\
&\quad +(1-\delta)\phi(0,c_A)\,(1-\phi\,(c_A,0)) \\
\Pr[(c_A+1,1)\,|\,(c_A,0)] &= \delta\,(1-\phi(c_A,0))\,(1-\phi(0,c_A))\,.
\end{aligned}$$

8. Suppose that (c_A, c_B) is such that $c_A = M$ and $c_B \notin \{0, M\}$. Then,

$$\begin{aligned}
\Pr[(M,c_B)\,|\,(M,c_B)] &= 1-\Omega, \\
\Pr[(M-1,c_B-1)\,|\,(M,c_B)] &= \phi\,(M,c_B)\,\phi(c_B,M) \\
&\quad +(1-\delta)\phi\,(M,c_B)\,(1-\phi(c_B,M)) \\
&\quad +(1-\delta)\phi(c_B,M)\,(1-\phi\,(M,c_B))\,, \\
\Pr[(M,c_B+1)\,|\,(M,c_B)] &= \delta\,(1-\phi\,(M,c_B))\,(1-\phi(c_B,M))\,.
\end{aligned}$$

9. Suppose that (c_A, c_B) is such that $c_A \notin \{0, M\}$ and $c_B = M$. Then,

$$\begin{aligned}
\Pr[(c_A,M)\,|\,(c_A,M)] &= 1-\Omega, \\
\Pr[(c_A-1,M-1)\,|\,(c_A,M)] &= \phi\,(c_A,M)\,\phi(M,c_A) \\
&\quad +(1-\delta)\phi\,(c_A,M)\,(1-\phi(M,c_A)) \\
&\quad +(1-\delta)\phi(M,c_A)\,(1-\phi\,(c_A,M))\,, \\
\Pr[(c_A+1,M)\,|\,(c_A,M)] &= \delta\,(1-\phi\,(c_A,M))\,(1-\phi(M,c_A))\,.
\end{aligned}$$

General Transition Matrix

Suppose that $\gamma \in (0, 1)$. Let V be the transition matrix for R&D competition/cartelization, and Z be the transition matrix for RJV-duplicate competition/cartelization. Then, the transition matrix, given γ and ψ is $(1 - \gamma)V + \gamma Z$.