**Tutoriel - Algorithme du transport - Tableaux**

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| T1. L’offre et la demande | | | | | | |
| Laboratoire L*i* | L1 | | L2 | L3 | | Total |
| Offre S*i* | 240 | | 160 | 260 | | 660 |
| Centre C*j* | C1 | C2 | C3 | C4 | C5 |  |
| Demande D*j* | 120 | 130 | 145 | 125 | 140 | 660 |

Exemple tiré de *Méthodes d'optimisation pour la gestion*; par Y. Nobert, R. Ouellet et R. Parent; 2008, Gaëtan Morin, éditeur; p. 293.

Voir aussi *La recherche opérationnelle, 3e édition*; par Y. Nobert, R. Ouellet et R. Parent; 2001, Gaëtan Morin, éditeur; page 358.

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| T2. Coûts unitaires de transport | | | | | |
|  | C1 | C2 | C3 | C4 | C5 |
| L1 | 1 | 8 | 1 | 5 | 4 |
| L2 | 5 | 5 | 3 | 6 | 7 |
| L3 | 2 | 9 | 5 | 9 | 8 |

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| T3. Tableau de transport (sans les données) | | | | | | | | | | |
| Origine | Destination | | | | | | | | | |
| C1 | | C2 | | C3 | | C4 | | C5 | |
| L1 |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |
| L2 |  |  |  |  |  |  |  |  |  |  |
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| L3 |  |  |  |  |  |  |  |  |  |  |
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| T4. Tableau de transport (avec offre et demande) | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | | C5 | |
| L1 |  |  |  |  |  |  |  |  | |  |  |  |
|  | |  | |  | |  | | |  | | 240 |
| L2 |  |  |  |  |  |  |  | |  |  |  |  |
|  | |  | |  | |  | | |  | | 160 |
| L3 |  |  |  |  |  |  |  | |  |  |  |  |
|  | |  | |  | |  | | |  | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | | 140 | | 660 |

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| T5. Tableau de transport | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 |  |
|  | |  | | |  | |  | |  | | 240 |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | |  | | |  | |  | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | |  | |  | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| T6*a*. Solution de base admissible initiale : tableau 0 | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 |  |
| **120** | | **120** | | |  | |  | |  | | 240 |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | | **10** | | | **145** | | **5** | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | | **120** | | **140** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| **T6*b*. Solution de base associée au tableau 0** |
| *x*11 = 120 et *x*12 = 120 et *x*22 = 10 et *x*23 = 145 et *x*24 = 5 et *x*34 = 120 et *x*35 = 140  *x*13 = *x*14 = *x*15 = *x*21 = *x*25 = *x*31 = *x*32 = *x*33 = 0 |

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| T7. Coût total de la solution initiale | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | Total |
| C1 | | C2 | | C3 | | C4 | | C5 | |
| L1 |  |  |  |  |  |  |  |  |  |  |  |
| **120** | | **960** | |  | |  | |  | | **1 080** |
| L2 |  |  |  |  |  |  |  |  |  |  |  |
|  | | **50** | | **435** | | **30** | |  | | **515** |
| L3 |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | | **1 080** | | **1 120** | | **2 200** |
| Total | **120** | | **1 010** | | **435** | | **1 110** | | **1 120** | | **3 795** |

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| **T8*a*. Heuristiques pous la construction d’une solution admissible initiale** |
| * La méthode du coin nord-ouest. * La méthode des coûts minimaux. * La méthode des pénalités. |

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| **T8*b*. La méthode du coin nord-ouest** |
| 1. Amorcer la méthode avec la case située dans le coin supérieur gauche du tableau de transport. 2. Attribuer le plus d'unités possible à la case courante. 3. Aller à une case adjacente à la case courante, en se déplaçant soit vers la droite, soit vers le bas. Revenir à l'étape 2.   La méthode s'arrête une fois effectuée l'attribution à la case située dans le coin inférieur droit. |

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| T9. Cycle de changement associé à la case hors base (1 ; 3) | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 |  |
| **120** | | **120−Δ** | | | **Δ** | |  | |  | | 240 |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | | **10+Δ** | | | **145−Δ** | | **5** | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | | **120** | | **140** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| T10. Tableau 0 : coûts marginaux | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 | -5 | 1 | -4 | 5 | -4 | 4 |  |
| **120** | | **120** | | |  | |  | |  | | 240 |
| L2 | 7 | 5 |  | 5 | |  | 3 |  | 6 | 2 | 7 |  |
|  | | **10** | | | **145** | | **5** | |  | | 160 |
| L3 | 1 | 2 | 1 | 9 | | -1 | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | | **120** | | **140** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| T11*a*. Tableau 0 et cycle de changement de la variable entrante | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 | -5 | 1 | -4 | 5 | -4 | 4 |  |
| **120** | | **120−Δ** | | | **Δ** | |  | |  | | 240 |
| L2 | 7 | 5 |  | 5 | |  | 3 |  | 6 | 2 | 7 |  |
|  | | **10+Δ** | | | **145−Δ** | | **5** | |  | | 160 |
| L3 | 1 | 2 | 1 | 9 | | -1 | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | | **120** | | **140** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |
| T11*b*. Cycle de changement associé à la case hors base (1 ; 5) | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 |  |
| **120** | | **120−Δ** | | |  | |  | | **Δ** | | 240 |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | | **10+Δ** | | | **145** | | **5−Δ** | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | | **120+Δ** | | **140−Δ** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| T12. Solution de base résultante : tableau 1 | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 |  |
| **120** | |  | | | **120** | |  | |  | | 240 |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | | **130** | | | **25** | | **5** | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | | **120** | | **140** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| T13. Tableau 1 et coûts marginaux | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 | 5 | | 8 |  | 1 | 1 | 5 | 1 | 4 |  |
| **120** | |  | | | **120** | |  | |  | | 240 |
| L2 | 2 | 5 |  | 5 | |  | 3 |  | 6 | 2 | 7 |  |
|  | | **130** | | | **25** | | **5** | |  | | 160 |
| L3 | -4 | 2 | 1 | 9 | | -1 | 5 |  | 9 |  | 8 |  |
|  | |  | | |  | | **120** | | **140** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| **T14*a*. Coût marginal  d’une variable *xij* dans une solution de base** |
| = *cij*  − (*ui* + *vj*) (1) |
| où  *cij*  : coût unitaire de la case (*i; j*)  *ui* et *vj*: valeurs des variables duales. |

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| **T14*b*. Coût marginal et test d’optimalité** |
| Le coût marginal d’une case hors base mesure l’impact sur le coût total *z* résultant du fait d’augmenter de 1 unité la quantité expédiée associée à cette case tout en laissant nulles les quantités *xij* associées aux autres cases hors base du tableau. |
| **Test d’optimalité :** Si les coûts marginaux de toutes les cases hors base sont ≥ 0, il n’est pas possible de diminuer la valeur de *z* et la solution de base courante est optimale. |

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| **T14*c*. Test d’optimalité** |
| * Si les coûts marginaux des cases hors base sont tous ≥ 0, il n’est pas possible de diminuer la valeur du coût total *z* et la solution de base courante est optimale. * Si une ou plusieurs cases hors base admettent un coût marginal négatif, on construit une nouvelle solution de base en choisissant une des cases hors base dont le coût marginal est négatif et en la forçant à devenir case de base. Cette case dont le statut est modifié est dite *case entrante*. |

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| **T14*d*. Principe général pour trancher les cas d’égalité** |
| Toutes autres choses étant égales par ailleurs, une case dont le coût unitaire est moins élevé a priorité dans la base*.* |

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| **T15*a*. Calcul des valeurs des variables duales** |
| Pour toute variable de base *xij*, *cij*  = *ui* + *vj*. (2) |

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| **T15*b*. Calcul des valeurs des variables duales** |
| Nombre de variables de base = *m + n* − 1 = 3 + 5 − 1 = 7  Nombre de variables duales = Nombre de  rangées = *m + n* = 3 + 5 = 8 |

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| **T16*a*. Calcul des valeurs des variables duales** |
| Pour toute variable de base *xij*, *cij*  = *ui* + *vj*. (2)  En particulier,  *c*11 = *u*1 + *v*1  1 = 0 + *v*1. |

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| **T16*b*. Calcul des valeurs des variables duales** |
| Pour toute variable de base *xij*, *cij*  = *ui* + *vj*. (2)  En particulier,  *c*12 = *u*1 + *v*2  8 = 0 + *v*2. |
| **T16*c*. Calcul des valeurs des variables duales** |
| Pour toute variable de base *xij*, *cij*  = *ui* + *vj*. (2)  En particulier,  *c*22 = *u*2 + *v*2  5 = *u*2 + 8  *u*2 = 5 – 8 = -3. |

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| **T17. Calcul des valeurs des variables duales** |
| Pour toute variable de base *xij*, *cij*  = *ui* + *vj*. (2)  En particulier,  *c*35 = *u*3 + *v*5  8 = *u*3 + *v*5. |

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| **T18*a*. Coût marginal  d’une variable hors base *xij*** |
| = *cij*  − (*ui* + *vj*) (1) |
| = *c*13 − (*u*1 + *v*3)  = 1 − (0 + 6)  = -5 |

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| **T18*b*. Coût marginal  d’une variable hors base *xij*** |
| = *cij*  − (*ui* + *vj*) (1) |
| = *c*21 − (*u*2 + *v*1)  = 5 − (-3 + 1)  = 5 − (-2) = 5 + 2 = 7 |

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| **T19. Valeur maximale de la variable entrante** |
| * d’où   en résumé :   * d’où |

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| **T20. Coût total du tableau 1** |
| Diminution du coût total = 5 × 120 = 600  (Coût total du tableau 1) = (Coût total du tableau 0) − Diminution  = 3 795 − 600  = 3 195 |

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| **T21. Calcul des valeurs des variables duales** |
| Pour toute variable de base *xij*, *cij*  = *ui* + *vj.* (2)  En particulier,  *c*11 = *u*1 + *v*1 et *c*13 = *u*1 + *v*3  1 = 0 + *v*1 et 1 = 0 + *v*3. |

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| **T22. Calcul des valeurs des variables duales** |
| Pour toute variable de base *xij*, *cij*  = *ui* + *vj*. (2)  En particulier,  *c*23 = *u*2 + *v*3  3 = *u*2 + 1. |

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| **T23. Coût marginal  d’une variable hors base *xij*** |
| = *cij*  − (*ui* + *vj*) (1) |
| = *c*31 − (*u*3 + *v*1)  = 2 − (1+ 5)  = -4 |

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| **T24. Solution de base associée au tableau 3 : solution optimale** |
| *x*13 = 145 et *x*15 = 95 et *x*22 = 130 et *x*24 = 30 et *x*31 = 120 et *x*34 = 95 et *x*35 = 45  *x*11 = *x*12 = *x*14 = *x*21 = *x*23 = *x*25 = *x*32 = *x*33 = 0. |

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| **T25. Coût total de la solution de base associée au tableau 3** |
| *x*13 = 145 et *x*15 = 95 et *x*22 = 130 et *x*24 = 30 et *x*31 = 120 et *x*34 = 95 et *x*35 = 45  *x*11 = *x*12 = *x*14 = *x*21 = *x*23 = *x*25 = *x*32 = *x*33 = 0. |
| Coût total = (1 × 145) + (4 × 95) + (5 × 130) + … + (8 × 45) = 2 810 |

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| T26. Un problème non équilibré. L’offre et la demande | | | | | | |
| Laboratoire L*i* | L1 | | L2 | L3 | | Total |
| Offre S*i* | 250 | | 160 | 260 | | 670 |
| Centre C*j* | C1 | C2 | C3 | C4 | C5 |  |
| Demande D*j* | 120 | 130 | 145 | 125 | 140 | 660 |

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| T27. Problème non équilibré - Ajout d’une destination fictive | | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | | C6 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  |  |  |
|  | |  | |  | |  | |  | | **7** | | 250 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  |  |  |
|  | |  | |  | |  | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  |  |  |
|  | |  | |  | |  | |  | |  | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 10 | | 670 |

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| T28. Problème non équilibré - Tableau de transport | | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | | C6 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 0 |  |
|  | |  | |  | |  | |  | |  | | 250 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 0 |  |
|  | |  | |  | |  | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  | 0 |  |
|  | |  | |  | |  | |  | |  | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 10 | | 670 |

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| T29. Problème non équilibré : début de la méthode du coin nord-ouest | | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | | C6 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 0 |  |
| **120** | | **130** | |  | |  | |  | |  | | 250 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 0 |  |
|  | |  | |  | |  | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  | 0 |  |
|  | |  | |  | |  | |  | |  | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 10 | | 670 |

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| T30. Problème non équilibré : une solution qui n’est pas une solution de base | | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | | C6 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 0 |  |
| **120** | | **130** | |  | |  | |  | |  | | 250 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 0 |  |
|  | |  | | **145** | | **15** | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  | 0 |  |
|  | |  | |  | | **110** | | **140** | | **10** | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 10 | | 670 |

Nombre théorique de variables de base = *m + n* − 1 = 3 + 6 − 1 = 8

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| T31. Problème non équilibré : une solution qui n’est pas une solution de base | | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | | C6 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 0 |  |
| **120 − Δ** | | **130** | |  | | **Δ** | |  | |  | | 250 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 0 |  |
|  | |  | | **145** | | **15** | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  | 0 |  |
|  | |  | |  | | **110** | | **140** | | **10** | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 10 | | 670 |

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| T32. Problème non équilibré : 3e attribution | | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | | C6 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 0 |  |
| **120** | | **130** | | **0** | |  | |  | |  | | 250 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 0 |  |
|  | |  | |  | |  | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  | 0 |  |
|  | |  | |  | |  | |  | |  | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 10 | | 670 |

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| **T33*a*. Solution de base dégénérée** |
| Une solution de base dégénérée estune solution de base dans laquelle une ou plusieurs variables de base prennent la valeur 0. |

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| T33*b*. Problème non équilibré : une solution de base dégénérée | | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | | C6 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 0 |  |
| **120** | | **130** | | **0** | |  | |  | |  | | 250 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 0 |  |
|  | |  | | **145** | | **15** | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  | 0 |  |
|  | |  | |  | | **110** | | **140** | | **10** | | 260 |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 10 | | 670 |

Nombre de variables de base = *m + n* − 1 = 3 + 6 − 1 = 8

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| T34. Tableau 3 et coûts marginaux | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 | 3 | 1 | 4 | | 8 |  | 1 | 0 | 5 |  | 4 |  |
|  | |  | | | **145** | |  | | **95** | | 240 |
| L2 | 6 | 5 |  | 5 | | 1 | 3 |  | 6 | 2 | 7 |  |
|  | | **130** | | |  | | **30** | |  | | 160 |
| L3 |  | 2 | 1 | 9 | | 0 | 5 |  | 9 |  | 8 |  |
| **120** | |  | | |  | | **95** | | **45** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| T35. Tableau 3 et cycle de changement de la case (1 ;4) | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 | 3 | 1 | 4 | | 8 |  | 1 | 0 | 5 |  | 4 |  |
|  | |  | | | **145** | | **Δ** | | **95 − Δ** | | 240 |
| L2 | 6 | 5 |  | 5 | | 1 | 3 |  | 6 | 2 | 7 |  |
|  | | **130** | | |  | | **30** | |  | | 160 |
| L3 |  | 2 | 1 | 9 | | 0 | 5 |  | 9 |  | 8 |  |
| **120** | |  | | |  | | **95 − Δ** | | **45 + Δ** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| T36. Tableau optimal dégénéré | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 | 3 | 1 | 0 | | 4 |  | 1 | 0 | 5 |  | 4 |  |
|  | |  | | | **145** | |  | | **95** | | 240 |
| L2 | 6 | 5 |  | 5 | | 1 | 3 |  | 6 | 2 | 7 |  |
|  | | **130** | | |  | | **30** | |  | | 160 |
| L3 |  | 2 | 1 | 9 | | 1 | 6 |  | 9 |  | 8 |  |
| **120** | |  | | |  | | **0** | | **45** | | 165 |
| Demande | 120 | | 130 | | | 145 | | 30 | | 140 | | 565 |

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| T37. Tableau optimal dégénéré | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 | 3 | 1 | 0 | | 4 |  | 1 | 0 | 5 |  | 4 |  |
|  | |  | | | **145** | | **Δ** | | **95 − Δ** | | 240 |
| L2 | 6 | 5 |  | 5 | | 1 | 3 |  | 6 | 2 | 7 |  |
|  | | **130** | | |  | | **30** | |  | | 160 |
| L3 |  | 2 | 1 | 9 | | 1 | 6 |  | 9 |  | 8 |  |
| **120** | |  | | |  | | **0 − Δ** | | **45 + Δ** | | 165 |
| Demande | 120 | | 130 | | | 145 | | 30 | | 140 | | 565 |

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| **T38. Critères pour décider de l’unicité de la solution optimale** |
| Dans un *tableau optimal* associé à un problème de transport :   * Si les coûts marginaux des variables hors base du tableau sont tous différents de 0, la solution de base associée au tableau est l’unique solution optimale du problème. * Si une variable hors base *xij* admet un coût marginal  nul et si cette variable, lorsque considérée comme variable entrante, peut prendre une valeur positive, alors le problème possède plusieurs solutions optimales . * Enfin, si certaines variables hors base admettent un coût marginal nul et si toutes ces variables sont limitées à la valeur 0 lorsque considérées comme variables entrantes, alors on ne peut conclure ni à l’unicité, ni à la multiplicité des solutions optimales à partir du tableau considéré. Il faudra effectuer une ou plusieurs itérations en prenant comme variables entrantes des variables hors base de coût marginal nul pour déterminer si le problème possède une seule ou plusieurs solutions optimales. |

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| T39. Méthode des coûts minimaux – 1re attribution | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 | 95  ~~240~~ |
|  | |  | | | **145** | |  | |  | |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | |  | | | **\*** | |  | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
|  | |  | | | \* | |  | |  | | 260 |
| Demande | 120 | | 130 | | | ~~145~~ 0 | | 125 | | 140 | | 660 |

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| T40. Méthode des coûts minimaux – 2e attribution | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 | 0  ~~95~~  ~~240~~ |
| **95** | | **\*** | | | **145** | | **\*** | | **\*** | |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | |  | | | **\*** | |  | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
|  | |  | | | \* | |  | |  | | 260 |
| Demande | ~~120~~ 25 | | 130 | | | ~~145~~ 0 | | 125 | | 140 | | 660 |

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| T41. Méthode des coûts minimaux – 3e attribution | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 | 0  ~~95~~  ~~240~~ |
| **95** | | **\*** | | | **145** | | **\*** | | **\*** | |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
| \* | |  | | | **\*** | |  | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 | 235  ~~260~~ |
| **25** | |  | | | \* | |  | |  | |
| Demande | ~~120~~ ~~25~~ 0 | | 130 | | | ~~145~~ 0 | | 125 | | 140 | | 660 |

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| T42*a*. Méthode des coûts minimaux – Solution de base résultante | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | | C2 | | C3 | | C4 | | C5 | |
| L1 |  | | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  |
| **95** | | |  | | **145** | |  | |  | | 240 |
| L2 |  | 5 | |  | 5 |  | 3 |  | 6 |  | 7 |  |
|  | | | **130** | |  | | **30** | |  | | 160 |
| L3 |  | 2 | |  | 9 |  | 5 |  | 9 |  | 8 |  |
| **25** | | |  | |  | | **95** | | **140** | | 260 |
| Demande | 120 | | | 130 | | 145 | | 125 | | 140 | | 660 |

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| T42*b*. Méthode des coûts minimaux – Avant la dernière attribution | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 | 0  ~~95~~  ~~240~~ |
| **95** | | **\*** | | | **145** | | **\*** | | **\*** | |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 | 0  ~~30~~  160 |
| \* | | **130** | | | **\*** | | **30** | | \* | |
| L3 |  | 2 |  | 9 | |  | 5 |  |  |  | 8 | 95  ~~235~~  ~~260~~ |
| **25** | | \* | | | \* | |  | | **140** | |
| Demande | ~~120~~ ~~25~~ 0 | | ~~130~~ 0 | | | ~~145~~ 0 | | ~~125~~ 95 | | ~~140~~ 0 | | 660 |

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| **T43*b*. Pénalité.** |
| La ***pénalité*** associée à une rangée est la différence, en valeur absolue, entre les deux coûts unitaires minimaux des cases disponibles apparaissant dans cette rangée. |

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| **T43*c*. Méthode des pénalités.** |
| On sélectionne la case où effectuer la prochaine attribution selon la règle hiérarchique suivante : on retient parmi les cases disponibles   * celles dont la rangée est associée à la pénalité la plus élevée; * celles de coût unitaire minimal; * celles auxquelles on peut attribuer la valeur maximale.   S’il reste encore plus d’une case candidate, le choix se fait alors de façon aléatoire parmi les candidates encore en lice. |

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| T44*a*. Méthode des pénalités – Avant la 1re attribution | | | | | | | | | | |  |  |
| Origine | Destination | | | | | | | | | | Offre |  |
| C1 | | C2 | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 0 |
|  | |  | |  | |  | |  | | 240 |  |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 2 |
|  | |  | |  | |  | |  | | 160 |  |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 |  | 3 |
|  | |  | |  | |  | |  | | 260 |  |
| Demande | 120 | | 130 | | 145 | | 125 | | 140 | | 660 |  |
|  | 1 | | 3 | | 2 | | 1 | | 3 | |  |  |

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| T44*b*. Méthode des pénalités – 1re attribution | | | | | | | | | | |  |
| Origine | Destination | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  |
| \* | |  | |  | |  | |  | | 240 |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  |
| \* | |  | |  | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 | 140 |
| **120** | |  | |  | |  | |  | | ~~260~~ |
| Demande | ~~120~~ 0 | | 130 | | 145 | | 125 | | 140 | | 660 |

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| T45*a*. Méthode des pénalités – Avant la 2e attribution | | | | | | | | | | |  |  |
| Origine | Destination | | | | | | | | | | Offre |  |
| C1 | | C2 | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 |  | 3 |
| \* | |  | |  | |  | |  | | 240 |  |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  | 2 |
| \* | |  | |  | |  | |  | | 160 |  |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 | 140 | 3 |
| **120** | |  | |  | |  | |  | | ~~260~~ |  |
| Demande | ~~120~~ 0 | | 130 | | 145 | | 125 | | 140 | | 660 |  |
|  | \* | | 3 | | 2 | | 1 | | 3 | |  |  |

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| T45*b*. Méthode des pénalités – 2e attribution | | | | | | | | | | |  |
| Origine | Destination | | | | | | | | | | Offre |
| C1 | | C2 | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 | 95 |
| \* | |  | | **145** | |  | |  | | ~~240~~ |
| L2 |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 |  |
| \* | |  | | \* | |  | |  | | 160 |
| L3 |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 | 140 |
| **120** | |  | | \* | |  | |  | | ~~260~~ |
| Demande | ~~120~~ 0 | | 130 | | ~~145~~ 0 | | 125 | | 140 | | 660 |

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| T46. Méthode des pénalités – Solution de base résultante | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | C2 | | | C3 | | C4 | | C5 | |
| L1 |  | 1 |  | | 8 |  | 1 |  | 5 |  | 4 |  |
|  | |  | | | **145** | |  | | **95** | | 240 |
| L2 |  | 5 |  | 5 | |  | 3 |  | 6 |  | 7 |  |
|  | | **130** | | |  | | **30** | |  | | 160 |
| L3 |  | 2 |  | 9 | |  | 5 |  | 9 |  | 8 |  |
| **120** | |  | | |  | | **95** | | **45** | | 260 |
| Demande | 120 | | 130 | | | 145 | | 125 | | 140 | | 660 |

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| **T47. Comparaison des heuristiques pour l’exemple Sporcau.** | | |
| Solution initiale de l’exemple Sporcau selon la méthode : | | |
| * du coin nord-ouest : | tableau 0 | coût total = 3 795; |
| * des coûts minimaux : | tableau 2 | coût total = 3 095; |
| * des pénalités : | tableau 3 | coût total = 2 810. |

**T\*\*. Calcul des valeurs des variables duales et des coûts marginaux**

|  | *v*1 = | | *v*2 = | | *v*3 = | | *v*4 = | | *v*5 = | | S*i* |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *u*1 = |  | 1 |  | 8 |  | 1 |  | 5 |  | 4 | 240 |
|  |  | |  | |  | |  | |  | |
| *u*2 = |  | 5 |  | 5 |  | 3 |  | 6 |  | 7 | 160 |
|  |  | |  | |  | |  | |  | |
| *u*3 = |  | 2 |  | 9 |  | 5 |  | 9 |  | 8 | 260 |
|  |  | |  | |  | |  | |  | |
| D*j* | 120 | | 130 | | 145 | | 125 | | 140 | | 660 |

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| T\*\*\*. Tableau 2 et coûts marginaux | | | | | | | | | | | | |
| Origine | Destination | | | | | | | | | | | Offre |
| C1 | | | C2 | | C3 | | C4 | | C5 | |
| L1 |  | | 1 | 1 | 8 |  | 1 | -3 | 5 | -3 | 4 |  |
| **95** | | |  | | **145** | |  | |  | | 240 |
| L2 | 6 | 5 | |  | 5 | 4 | 3 |  | 6 | 2 | 7 |  |
|  | | | **130** | |  | | **30** | |  | | 160 |
| L3 |  | 2 | | 1 | 9 | 3 | 5 |  | 9 |  | 8 |  |
| **25** | | |  | |  | | **95** | | **140** | | 260 |
| Demande | 120 | | | 130 | | 145 | | 125 | | 140 | | 660 |